Empirical test of structural model under time-varying volatility

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ABSTRACT

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In trying to explain the "Credit Spread Puzzle", we empirically examine two competing structural models: the Leland (1994b) constant volatility model and the Perrakis and Zhong (2013) Constant Elasticity of Variance (CEV) model. We use the Leland model as our benchmark and hypothesize that the CEV model under state-dependent volatility will outperform it. For our estimation, we incorporate firm level time series data from different markets. Our sample covers the period from 2001 to 2011. We apply the General Method of Moment (GMM) for our estimation of the parameters of the diffusion process for the Leland and CEV models respectively. In our results, we document on average a significantly negative beta, the elasticity parameter in the Perrakis and Zhong CEV model. More importantly, we find that the CEV model can fit the historical data much better than the constant volatility Leland (1994b) model across all maturities, suggesting that the state-dependent volatility can explain the "Credit Spread Puzzle" to some extent.

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1. Introduction

Pricing debt as a contingent claim on the firm's value is an approach known as a *structural model* of the firm, adopted from the option pricing domain. Following the Black and Scholes (1973) theory of option pricing, Merton (1974) implemented this approach in his pioneer work of bond pricing and risk structure of interest rates. In this approach, the firm asset value is treated as the underlying asset and the balance sheet items such as liability and equity play the role of contingent claims that can be valued by methods adapted from option valuation. Because of its detailed results, the structural model has drawn a lot of attention from academics and practitioners. Earlier studies have tackled, among others, the issues of debt pricing, credit risk, and optimal capital structure by applying this method. These structural models of the firms are distinct from the other major class of bond pricing models known as *reduced form* models. In reduced form models the underlying asset is no longer the unlevered firm value. Instead, observable variables such as equity returns and equity value take the place of underlying assets. Therefore, reduced form models do not tackle the optimal capital structure as structural models do.

The first study by Merton (1974) imposed several assumptions and restrictions to the model which may seem a little bit unrealistic by now. These restrictions include debt composed of zero coupon discount bonds, no transaction costs, no taxes, no bankruptcy cost, fully liquid markes and default that can only happen at the time of debt maturity. The firm value V was set to follow a simple diffusion process. Thus, we can treat any security of the firm as a contingent claim on the underlying asset, the firm value. The closed-form solution was derived under this framework. Based on this pioneer model, Black and Cox (1976) proposed a new model which relaxed some of the assumptions. They allow the default to happen before the maturity and make the default boundary depend on certain types of bond indenture provisions. In particular, they examined the effect of three types of provision: safety covenants, subordination agreements and restrictions on the financing of interest and dividend payments. Their conclusion is that these provisions would certainly increase the value of bonds and may affect the behaviour of the firm's securities. Also, they mentioned the possible effects of the introduction of bankruptcy cost and taxes, the time-varying volatility and the presence of a jump process. Although these models were adopted in many subsequent studies, the criticisms never disappeared. One of the most important such criticisms is the empirical result found by many researchers that the credit spreads predicted by those models were much smaller than actually observed credit spreads.[1] This phenomenon was termed the "Credit Spread Puzzle" by Huang and Huang (2003). Several subsequent studies have tried to explain this phenomenon from different aspects. In Longstaff and Schwartz (1995) the authors proposed a two-factor model which incorporated interest rate risk. They derived a closed-form solution for risky coupon bond and debt value, distinguishing their work from that of others who also took into consideration interest-rate risk. They concluded that the interest rate can affect the valuation of firm securities through its correlation with firm value. Following Longstaff and Schwartz (1995), Collin-Dufresne and Glodstein (2001) adopted this same two-factor framework to allow the interest rate to follow a stochastic process. However, they relaxed the assumption of constant default boundary while still keeping it exogenous. They argued that there was a target level of leverage for each individual firm or the firms in a certain industry, a stationary leverage ratio. They used a mean-reverting default threshold to represent this feature. Most importantly, in their work, they developed an exact solution for the Fortet equation of the first passage time to default under a multi-dimensional diffusion framework, which in Longstaff and Schwartz (1995) is only found by an approximation of the true solution. Their model predicts credit risk more consistent with the observed credit spread. Huang and Zhou (2008) show that the Collin-Dufresne and Glodstein (2001) model was the only one that survives their empirical test.

[1] Longstaff and Schwartz (1995)

Leland (1994a, b) and Leland and Toft (LT 1996), introduced the endogenous default boundary for infinite maturity debt and finite maturity debt respectively. They also incorporated in their model the Modigliani and Miller (MM) theorems by introducing the tax benefit and bankruptcy cost of debt into their model. They derived closed-form solutions for corporate debt value, firm value, equity value and the endogenous default boundary. Based on Leland (1994b), Perrakis and Zhong (2013) proposed a new structural model which incorporated time-varying volatility. Unlike the working paper of Elkamhi, Ericsson and Jiang (2011), which also introduced time-varying stochastic volatility into their structural model, Perrakis and Zhong (2013) used constant elasticity variance (CEV), a one-dimensional asset dynamic that significantly reduces the complexity of derivation and calculation of the model. Their conjecture is that the time-varying volatility will explain the "Credit Spread Puzzle" to some extent. Our objective in this paper is to test their conjecture empirically.

We adopt the framework and method from Huang and Zhou (2008). In their work, these authors used Credit Default Swap (CDS) market information because compared with corporate bond spreads the CDS spreads are relatively more pure for default risk pricing because of better liquidity in their respective markets. They also used the whole term-structure of the CDS spreads that can make the pricing error of the model more efficient. To solve this over-identified system, the General Method of Moments (GMM) is implemented here for the parameter estimation.[2] Unlike Huang and Zhou (2008), which examined five classic structural models and compared their performance, our estimation goal is more specific here. We like to examine whether the introduction of time-varying volatility would explain the well-known "Credit Spread Puzzle".

[2] In Duan (1994), Maximum Likelihood Estimation (MLE) was introduced as a superior method to estimate the parameters for a unobservable asset value process. In Ericsson and Reneby (2005), the authors also demonstrated the strength of MLE in this kind of estimation work compared to the previous method. However, in Huang and Zhou (2008), the term-structure of CDS spreads plus the equity volatility made the system an overidentified one since normally there are just two or three parameters to be estimated. So GMM estimation was implemented here to incorporate all the information carried in the moment conditions. We adopted this same econometric method, following also Elkamhi, Ericsson and Jiang (2011).

Therefore, we only compared two models directly. One is Leland (1994b), with constant volatility, finite maturity and a closed-form solution; the second one is Perrakis and Zhong (2013) which adopts CEV asset dynamic for firm value and make the Leland (1994b) a special case, while still having a closed-form solution.

We collected 10 years firm level time series data for the estimation. Then we fitted all the available historical data into these two competing models. Those data include historical CDS spreads, firm financial statement data, equity market data, historical term structure of interest rate, and implied equity volatility from option market. Since σ and β are the only parameters we are interested, all the data from the different markets build up an over-identified model. Therefore, we applied the General Method of Moment (GMM) to empirically estimate our parameters. In our results, we document that the CEV model with time-dependent volatility outperforms the Leland (1994b) constant volatility model in fitting across all maturities.

To close our introduction and literature review, we discuss briefly the reduced form models. Even if our scope in this paper does not cover these models, they are an alternative strand of models for debt valuation. Similar with structural models, reduced form models also allow their primary asset dynamics to follow the diffusion or jump diffusion process. Reduced form models have their own advantages by using observable variables as their underlying asset instead of the unobservable unlevered firm value. However, reduced form models have the disadvantage of lacking the link between the default process and the capital structure, or the first time that asset value falls below a certain level. Due to this reason, reduced form models will not be examined and tested in this paper.[3]

The rest of this paper is structured as follow. Section 2 will briefly review the two models that we will compare in our estimation. Section 3 will discuss our econometric method and our empirical

[3] See Jarrow and Turnbull (1995), Duffie and Singleton (1999) and Duffie and Lando (2001) for more details on reduced form models.

estimation techniques in details. Section 4 will present our data cleaning and construction. Section 5 will be the analysis of our empirical results. Finally, Section 6 concludes.

2. Review of structural models

In this section, we will review several important structural models which have appeared in the literature since the Merton model. They are: Merton (1974), Black and Cox (1976), Longstaff and Schwartz (1995), Collin-Dufresne and Goldstein (2001), Leland (1994b), and Perrakis and Zhong (2013). The structural models that will be tested by our empirical work are the last two models: Leland (1994b) and Perrakis and Zhong (2013) constant elasticity of variance (CEV) model. Since the major concern of our empirical work is testing whether the introduction of time-varying volatility into the model can improve the fitting of the historical data, we set in these two models everything equal except for the parameter which represents the time variation of the volatility. In other words, we consider the Leland (1994b) model as a special case of Perrakis and Zhong (2013) CEV model when the elasticity parameter β is equal to zero. Therefore, we can get a straightforward result of the model performance by comparing the fitting of these two models.

2.1 The Merton (1974) model

The Merton (1974) model is the pioneer structural model, which considers securities of a firm as contingent claims on the underlying asset, firm value. This model has relatively strict restrictions such as: no transaction costs or taxes and bankruptcy costs, fully liquid market, zero coupon bond, unlimited borrowing and lending and the default can only happen at maturity. Unlike default in the subsequent barrier models, default in this model happens when the firm cannot pay the promised payment to the debt holder at maturity. Namely, when $V < B$, the firm will not make the payment to the debt holder and default, otherwise the equity holder will pay extra money.

Here V is the underlying asset or firm value and B is the promised payment to the debt holder at maturity. The firm value V is following a simple diffusion process:

$$dV = \mu V dt + \sigma V dW \tag{2.1}$$

Therefore, the market value of any security of the firm at any point of time can be written as a function of the value of the firm at that time, $Y = F(V,t)$. Applying Ito's lemma, we can get the diffusion process for debt and then the differential equation which must be satisfied by the value of the debt:

$$\frac{1}{2}\sigma^2 V^2 F_{VV} + rVF_V - rF - F_\tau = 0 \tag{2.2}$$

Subject to the condition:

$$F(V,0) = Min(V,B)$$

Then the value of the equity of the firm can be written as $f(V,t) = V - F(V,t)$, and it satisfies the following partial differential equation:

$$\frac{1}{2}\sigma^2 V^2 f_{VV} + rVf_V - rf - f_\tau = 0 \tag{2.3}$$

Subject to the condition:

$$f(V,0) = Max(0, V - B)$$

It is identical to a European call option with the firm value corresponding to the stock price and the payment B corresponds to the exercise price. Then we get directly the solution of the differential equation from the Black-Scholes option model:

$$f(V,\tau) = V\phi(x_1) - Be^{-r\tau}\phi(x_2) \qquad (2.4)$$

Where

$$\phi(x) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left[-\frac{1}{2}z^2\right]dz$$

And

$$x_1 \equiv \left\{\log\left[V/B\right] + \left(r + \frac{1}{2}\sigma^2\right)\tau\right\} / \sigma\sqrt{\tau}$$
$$x_2 \equiv x_1 - \sigma\sqrt{\tau}$$

From $f(V,t) = V - F(V,t)$, we can get the value of the debt as:

$$F(V,\tau) = Be^{-r\tau}\left\{\phi\left[h_2\left(d,\sigma^2\tau\right)\right] + \frac{1}{d}\phi\left[h_1\left(d,\sigma^2\tau\right)\right]\right\} \qquad (2.5)$$

Where

$$d \equiv Be^{-r\tau} / V$$
$$h_1\left(d,\sigma^2\tau\right) \equiv -\left[\frac{1}{2}\sigma^2\tau - \log\left(d\right)\right]\Big/\sigma\sqrt{\tau}$$
$$h_2\left(d,\sigma^2\tau\right) \equiv -\left[\frac{1}{2}\sigma^2\tau - \log\left(d\right)\right]\Big/\sigma\sqrt{\tau}$$

2.2 The Black and Cox (1976) model

After Merton (1974), Black and Cox relaxed some of that model's assumptions and examined the effect of certain types of bond indentures which are encountered in practice. Namely, they examined three kinds of bond indentures: safety covenants, subordination arrangements and restrictions on the financing of interest and dividend payments. Their model has the following assumptions: fully liquid market; no transaction cost, tax and bankruptcy cost; unlimited borrowing and lending with identical interest rate; and the value of the firm follow a diffusion process. However, default can happen before maturity when the firm value hits a certain boundary.

2.2.1 Safety covenants

Safety covenants are contractual provisions that give the debt holder the right to force a bankruptcy or firm reorganization before maturity if the firm is doing poorly by certain standards which are described in the covenants. One of these standards is that the firm omit the interest payment to the debt holder. However, the authors argued that if the equity holders are allowed to sell assets to fulfill the requirement, then this provision is not effective. Therefore, they made the safety covenants as follow: if the firm asset value falls below a certain level which was decided in the covenants, then the debt holder has the right to force a bankruptcy or reorganization. Thus, the value of bond F will satisfy the following differential equation:

$$\frac{1}{2}\sigma^2 V^2 F_{VV} + (r-\alpha)VF_V - rF + F_t = 0 \tag{2.6}$$

Subject to condition

$$\begin{aligned} &F(V,T) = Min(V,P) \\ &F(Ce^{-\gamma(T-t)},t) = Ce^{-\gamma(T-t)} \end{aligned}$$

Where P is the promised payment to the debt holder, α is the proportion of dividend the equity holder can receive continuously and $Ce^{-\gamma(T-t)}$ is the time-depended bankruptcy level.

Similarly, the value of stock has to satisfy the following differential equation:

$$\frac{1}{2}\sigma^2 V^2 f_{VV} + (r-\alpha)Vf_V - rf + f_t + \alpha V = 0 \tag{2.7}$$

Subject to the conditions

$$f(V,T) = Max(V-P,0)$$
$$f(Ce^{-\gamma(T-t)},t) = 0$$

Solving the differential equation, the authors got the closed form solution for debt under the safety covenants as:

$$\begin{aligned} F(V,t) = {} & Pe^{-r(T-t)}[N(Z_1) - y^{2\theta-2}N(Z_2)] + Ve^{-\alpha(T-t)}[N(Z_3) + y^{2\theta}N(Z_4) + y^{\theta+\zeta}e^{\alpha(T-t)}N(Z_5) \\ & + y^{\theta-\zeta}e^{\alpha(T-t)}N(Z_6) - y^{\theta-\eta}N(Z_7) - y^{\theta-\eta}N(Z_8)] \end{aligned} \tag{2.8}$$

Where

$$y = Ce^{-\gamma(T-t)}/V,\ \theta = (r-\alpha-\gamma+\frac{1}{2}\sigma^2)/\sigma^2,\ \zeta = \sqrt{\delta}/\sigma^2$$

$$\eta = \sqrt{\delta - 2\sigma^2\alpha}/\sigma^2,\ \delta = (r-\alpha-\gamma+\frac{1}{2}\sigma^2)^2 + 2\sigma^2(r-\gamma)$$

$$Z_1 = [Ln(V) - Ln(P) + (r-\alpha-\frac{1}{2}\sigma^2)(T-t)]/\sqrt{\sigma^2(T-t)}$$

$$Z_2 = [Ln(V) - Ln(P) + 2Ln(y) + (r-\alpha-\frac{1}{2}\sigma^2)(T-t)]/\sqrt{\sigma^2(T-t)}$$

$$Z_3 = [Ln(P) - Ln(V) - (r-\alpha+\frac{1}{2}\sigma^2)(T-t)]/\sqrt{\sigma^2(T-t)}$$

$$Z_4 = [Ln(V) - Ln(P) + 2Ln(y) + (r-\alpha+\frac{1}{2}\sigma^2)(T-t)]/\sqrt{\sigma^2(T-t)}$$

$$Z_5 = [Ln(y) + \zeta\sigma^2(T-t)]/\sqrt{\sigma^2(T-t)},\ Z_6 = [Ln(y) - \zeta\sigma^2(T-t)]/\sqrt{\sigma^2(T-t)}$$

$$Z_7 = [Ln(y) + \eta\sigma^2(T-t)]/\sqrt{\sigma^2(T-t)},\ Z_8 = [Ln(y) - \eta\sigma^2(T-t)]/\sqrt{\sigma^2(T-t)}$$

2.2.2 Subordination arrangements

The second kind of bond indenture provision is a subordination arrangement. It means that there exist two kinds of debt holders, senior debt holder and junior debt holder. Junior debt holders are subordinated to senior debt holders: they can get paid only after the promised payments to senior debt holders have been fully fulfilled at maturity. Suppose the payments to senior and junior debt holder are P and Q respectively. The author argued that the value for senior debt is the same as the value of debt in safety covenants provisions, and the value of junior debt is given by the following expression:

$$J(V,t)=\begin{cases} F(V,t;P+Q,\rho Pe^{-r(T-t)})-F(V,t;P,\rho Pe^{-r(T-t)}), & if\ \rho<1 \\ F(V,t;P+Q,\rho Pe^{-r(T-t)})-Pe^{-r(T-t)}, & if\ 1\le\rho\le\dfrac{P+Q}{P} \\ Qe^{-r(T-t)}, & if\ \rho\ge\dfrac{P+Q}{P} \end{cases} \tag{2.9}$$

Where $F(V,t;P,\rho Pe^{-r(T-t)})$ denote the expression given in (2.8), and $\rho Pe^{-r(T-t)})$ is the safety covenants boundary.

2.2.3 Restrictions on the financing of interest and dividend payments

Under the third kind of bond indenture provisions, the author supposed that the firm has interest paying bonds outstanding. It must fulfill these payments to the bond holder periodically. Once the firm missed one of these interest payments, the bondholder will force a reorganization and take over the firm. However, in this model, raising money by selling part of the firm asset is totally forbidden. Therefore, the stockholder can only issue new securities to meet the requirement of the payments. But in some situation the stockholder may not be able to do this if the equity value after the payments would be less than the payments. The author argued that even if the stockholders offer an equity issue which will dilute their own interest, there might be no taker for the issuance. Therefore, it explained the observed fact that many firms end up with bankruptcy even if their asset value is still quite significant. On the other hand, if the firm issues new bond, the old bond holders must require that the new bond be subordinate bond. However, issuing a new junior bond at this situation would in fact help the senior bond holder and hurt the stockholder. Because issuing junior bond will make it more likely that interest payments will be missed and the bondholder will take over the firm. After this discussion, the authors stated that the value of security should satisfy the following equation:

$$\frac{1}{2}\sigma^2 V^2 F_{VV} + rVF_V - rF + C = 0 \tag{2.10}$$

Then, the solution can be obtained as follow:

$$F(V) = \frac{C}{r} - [(\frac{\alpha}{1+\alpha})^{\alpha} - (\frac{\alpha}{1+\alpha})^{\alpha+1}](\frac{C}{r})^{\alpha+1} V^{-\alpha} \tag{2.11}$$

Where

$$\alpha = 2r / \sigma^2$$

And C is the continual interest payment of the bond.

2.3 The Longstaff and Schwartz (1995) and Collin-Dufresne and Goldstein (2001) models

After Black and Cox (1976), two other important structural models were presented in the literature. One such structural model is the Longstaff and Schwartz model. The authors adopted most features from Merton (1974) and Black and Cox (1976), while gradually relaxing more assumptions and examining more variables that might affect the credit spread. As in the previous models, they also allow the firm value to follow a diffusion process, but with one step further:

they make the interests rate time varying and following a diffusion process itself. Then basically, this model is a two dimensional diffusion model rather than the previous one-dimensional model. Moreover, in their model, a bankruptcy cost was introduced. With ω percent bankruptcy cost, if default happens, the bond holder will only receive $(1\text{-}\omega)$ percent of the face value of the default boundary, making their model more realistic. And they also challenged the strict absolute priority rules which were discussed in the Black and Cox (1976) model. They argued that growing evidences shows that in realistic corporate restructuring, the absolute priority rules are frequently violated. Furthermore, the authors provided evidence supporting that the actual payments allocation among different debt holders might be affected by many other factors such as: firm size, bargaining power of the debt holder, the strength of ties between firm manager and stockholders. Despite all these improvements of their model, they still adopted the setup in Black and Cox (1976) that the default boundary is a prefixed level which will not change during the process. In general, their major contributions in this model as they announced in paper are two aspects: first, the introduction of time varying interests rate; second, the violation of strict absolute priority rules.

In their model, the asset dynamic is as follow:

$$\begin{aligned} dV &= \mu V dt + \sigma V dZ_1 \\ dr &= \left(\zeta - \beta r\right) dt + \eta dZ_2 \end{aligned} \tag{2.12}$$

Where Z_1 and Z_2 are standard winner process, μ, σ, ζ, β, and η are constants, and the asset dynamics of r are drawn from the Vasicek (1977) model.

They derive the following expression for the value of fixed rate debt in their model:

$$P(X,r,T) = D(r,T) - \omega D(r,T)Q(X,r,T) \tag{2.13}$$

Where

$$
\begin{aligned}
&Q(X,r,T,n)=\sum_{i=1}^{n} q_i \\
&q_1 = N(a_1) \\
&q_i = N(a_i)-\sum_{j=1}^{i-1} q_j N(b_{ij}), \quad i=2,3,\ldots,n \\
&a_i = \frac{-Ln(X)-M\left(iT/n,T\right)}{\sqrt{S\left(iT/n\right)}} \\
&b_{ij} = \frac{M\left(jT/n,T\right)-M\left(iT/n,T\right)}{\sqrt{S\left(iT/n\right)-S\left(jT/n\right)}}
\end{aligned}
$$

And where

$$
\begin{aligned}
M\left(t,T\right)=&\left(\frac{\alpha-\rho\sigma\eta}{\beta}-\frac{\eta^2}{\beta^2}-\frac{\sigma^2}{2}\right)t+\left(\frac{\rho\sigma\eta}{\beta^2}+\frac{\eta^2}{2\beta^3}\right)\exp\left(-\beta T\right)\left(\exp\left(\beta t\right)-1\right) \\
&+\left(\frac{r}{\beta}-\frac{\alpha}{\beta^2}+\frac{\eta^2}{\beta^3}\right)\left(1-\exp\left(-\beta t\right)\right)-\left(\frac{\eta^2}{2\beta^3}\right)\exp\left(-\beta T\right)\left(1-\exp\left(-\beta t\right)\right) \\
S\left(t\right)=&\left(\frac{\rho\sigma\eta}{\beta}+\frac{\eta^2}{\beta^2}+\sigma^2\right)t-\left(\frac{\rho\sigma\eta}{\beta^2}+\frac{\eta^2}{2\beta^3}\right)\left(1-\exp\left(-\beta t\right)\right)+\left(\frac{\eta^2}{2\beta^3}\right)\left(1-\exp\left(-2\beta t\right)\right)
\end{aligned}
$$

And $D(r,T)$ is the value of a riskless discount bond given by Vasicek (1977) and have the following form:

$$
D(r,T)=\exp\left(A\left(T\right)-B\left(T\right)r\right) \tag{2.14}
$$

Where

$$
\begin{aligned}
A\left(T\right)&=\left(\frac{\eta^2}{2\beta^2}-\frac{\alpha}{\beta}\right)T+\left(\frac{\eta^2}{\beta^3}-\frac{\alpha}{\beta^2}\right)\left(\exp\left(-\beta T\right)-1\right)-\left(\frac{\eta^2}{4\beta^3}\right)\left(\exp\left(-2\beta T\right)-1\right) \\
B\left(T\right)&=\frac{1-\exp\left(-\beta T\right)}{\beta}
\end{aligned}
$$

Then, the expression for the value of floating rate debt is given as:

$$F(X,r,\tau,T) = P(X,r,T)R(r,\tau,T) + \omega D(r,T)G(X,r,\tau,T) \tag{2.15}$$

Where

$$R(r,\tau,T) = r\exp\left(-\beta\tau\right) + \left(\frac{\alpha}{\beta} - \frac{\eta^2}{\beta^2}\right)\left(1-\exp\left(-\beta\tau\right)\right) + \left(\frac{\eta^2}{2\beta^2}\right)\exp\left(-\beta T\right)\left(\exp\left(\beta\tau\right) - \exp\left(-\beta\tau\right)\right)$$

$$G(X,r,\tau,T,n) = \sum_{i=1}^{n} q_i \frac{C\left(\tau, iT/n\right)}{S\left(iT/n\right)} M\left(iT/n, T\right)$$

And where

$$C\left(\tau,T\right) = \left(\frac{\rho\sigma\eta}{\beta} + \frac{\eta^2}{\beta^2}\right)\exp\left(-\beta\tau\right)\left(\exp\left(\beta Min\left(\tau,t\right)\right) - 1\right)$$
$$- \frac{\eta^2}{2\beta^2}\exp\left(-\beta\tau\right)\exp\left(-\beta t\right)\left(\exp\left(2\beta Min\left(\tau,t\right)\right) - 1\right)$$

After Longstaff and Schwartz (1995), Collin-Dufresne and Goldstein (2001) also proposed a two factor model with stochastic interest rate which took a step further in relaxing the constant capital structure assumption and allowing the firm to issue new debt in the future. They also argued that there exists a target leverage level for each firm or each kind of industry. Then, the process for firm leverage follows mean-reverting asset dynamics, which means the firm will issue more debt if its leverage is below the target and otherwise retire the debt. In the later empirical work of Huang and Zhou (2008), the Collin-Dufresne and Glodstein (2001) model was the only one that survived their empirical tests.

2.4 The Leland (1994b) model

Leland (1994a) derives the close form solutions for corporate debt and optimal capital structure under an infinite maturity framework. Moreover, it introduces the impact of tax shield and bankruptcy cost into the model for the first time. The Leland (1994b) model inherited most of its assumptions but with one very important modification: changing the maturity of bonds from infinity to arbitrary maturity. However, to keep the time-homogeneous cash flow feature of the debt, the new model allows a constant fraction of currently outstanding debt to be retired and replaced by newly-issued counterpart. Then the debt service cash flow will be constant as long as the firm is solvent. The author defined the rate at which the principal of debt is retired as the retirement rate *g*. We assume that at time $t=0$ the firm has total principal P, paying a constant total coupon rate C. As time goes by, the remaining value for this debt will be $e^{-gt}P$. The bondholder will receive the cash flow including coupon payments and fraction repayment of principal as $e^{-gt}\left(C+gP\right)$. Average debt maturity is given by:

$$M = \int_0^\infty t\left(ge^{-gt}\right)dt = \frac{1}{g} \tag{2.16}$$

As in the previous Merton (1974), Black and Cox (1976) and Leland (1994a) studies, the firm value V follows a diffusion asset dynamic with constant volatility:

$$\frac{dV}{V} = \mu\left(V,t\right)dt + \sigma dz \tag{2.17}$$

Where dz is standard Brownian motion. This process will continue endlessly as long as the firm is solvent. The default is triggered when V first touches V_B, the default boundary which is endogenously determined by a "Smooth Pasting" condition.

Under this setup, Leland (1994b) derived the closed form solution for corporate debt D, firm value v, equity value, E and endogenous default boundary V_B.

Corporate debt value D:

$$D=\frac{C+gP}{r+g}\left[1-\left(\frac{V}{V_B}\right)^{-y}\right]+(1-\alpha)V_B\left(\frac{V}{V_B}\right)^{-y} \tag{2.18}$$

Where

$$y=\frac{\left(r-\delta-0.5\sigma^2\right)+\left[\left(r-\delta-0.5\sigma^2\right)^2+2(g+r)\sigma^2\right]^{0.5}}{\sigma^2} \tag{2.19}$$

α is the fraction of value lost in the event of bankruptcy; δ is the proportional payout rate; $(1-\alpha)V_B$ is the total amount that bondholder will receive if default happens.

Firm value v :

$$v=V+TB-BC \tag{2.20}$$

It can be interpreted as the total value of the firm equals the unlevered firm value plus the value of tax benefit, minus the bankruptcy cost. Tax benefit and bankruptcy cost are as follows:

$$TB=(\tau C/r)\left[1-\left(\frac{V}{V_B}\right)^{-x}\right] \tag{2.21}$$

$$BC=\alpha V_B\left(\frac{V}{V_B}\right)^{-x} \tag{2.22}$$

Implying:

$$v=V+(\tau C/r)\left[1-\left(\frac{V}{V_B}\right)^{-x}\right]-\alpha V_B\left(\frac{V}{V_B}\right)^{-x} \tag{2.23}$$

Where x is given by equation (2.4) of y by setting g =0, another word, when the average maturity of debt is infinity.

Equity value E :

$$E = V + \left(\frac{\tau C}{r}\right)[1 - \left(\frac{V}{V_B}\right)^{-x}] - \alpha V_B \left(\frac{V}{V_B}\right)^{-x} - \left(\frac{C + gP}{r + g}\right)\left[1 - \left(\frac{V}{V_B}\right)^{-y}\right] - (1-\alpha)V_B\left(\frac{V}{V_B}\right)^{-y} \quad (2.24)$$

This value of equity is calculated as firm value minus debt value: $E = v - D$. v and D are given by equations (2.23) and (2.18).

Endogenous default boundary:

$$V_B = \frac{\left[\frac{(C+gP)y}{r+g} - \frac{\tau C x}{r}\right]}{1 + \alpha x + (1-\alpha) y} \quad (2.25)$$

This closed form solution for endogenous default boundary is derived by applying the "Smooth Pasting" condition.

Therefore, we can clearly see that under this framework of Leland (1994b), the very neat and intuitive closed form solution for all the balance sheet items we are interested in can be derived easily. Due to its simplicity of computation and straightforward intuition, we apply this model in our empirical estimation as a benchmark to compare with the CEV model. However, there is a clarification needed to be made: in our estimation, we used the KMV[4] default boundary instead of the endogenous default boundary to make our comparison of these two models more directly.

[4] Moody's KMV defines as trigger value $K = P_{Short} + 0.5 * P_{Long}$. Where P represent firm liability.

2.5 The Perrakis and Zhong (2013) CEV model

This CEV model is the main target of our empirical estimation and test. It was derived in Stylianos Perrakis and Rui Zhong's working paper "Structural Models of the Firm under State-Dependent Volatility: Theory and Empirical Evidence". The authors took the Leland (1994b) as their base case and introduced the time-varying volatility into the model. The major assumptions also follow the previous models: continuous coupon payment, finite maturity for debt, endogenous default boundary[5] and first passage time default. The unlevered firm value V following a diffusion process with state dependent volatility $\sigma\left(V^D\right)$ (Q-distribution)

$$\frac{dV}{V} = (r-q)dt + \sigma(V^D)dW^Q \tag{2.26}$$

Where r is the risk free rate; q is the payout rate of the asset, including coupon to debt-holder and dividend to shareholder; $\sigma(V^D)$ is the state dependent volatility; W is the standard Brownian motion. Then if we consider the bond maturity date T, and the first time firm value touches the boundary τ, then we will have the following asset dynamics:

$$\begin{cases} \dfrac{dV_t}{V_t} = (r-q)dt + \sigma(V^D)dW^Q, \text{ if } 0 < t < \tau < T \\ V_t = \min\{V_\tau, K\} \text{ if } 0 < \tau \le t < T \end{cases} \tag{2.27}$$

Where K is the default boundary. Once the firm value V passes this value for the first time, the default will be triggered and the debt-holder will receive $\left(1-\alpha\right)K$. Under the CEV model, the

[5] In our empirical estimation and test, we reduce this endogenous default boundary into a KMV pre-fixed default boundary as we did for the Leland (1994b) model to make the computation more efficient and comparison between these two models more directly.

state-dependent volatility then can be presented as $\sigma(V^D) = \theta V^\beta$. β, the elasticity of volatility, is the key parameter in the CEV model, which is also the key parameter in our estimation. When $\beta > 0(< 0)$ then the volatility is positively (negatively) correlated with firm value; when $\beta = 0$, the model is reduced to constant volatility as in Leland (1994b). In Perrakis and Zhong (2013), the authors adopt $\beta < 0$ without further restrictions. However, in this paper, our empirical results can give reader an idea of how exactly beta is distributed across our sample.

To make the debt maturity finite in this model, the authors also applied the continuous retirement of fraction of the debt principal and replaced it with newly issued equal amount of debt. The retirement rate is also g, and satisfies the same relationship as Leland (1994b) does in equation (2.16):

$$M = \int_0^\infty t\left(ge^{-gt}\right)dt = \frac{1}{g}$$

Under this framework, Perrakis and Zhong (2013) derived the close form solution for corporate debt value, firm value and equity value.

Corporate debt D:

$$D\left(V, K, g\right) = \frac{C + gP}{r + g}\left(1 - \frac{\phi_{r+g}(V)}{\phi_{r+g}(K)}\right) + (1-\alpha)K\frac{\phi_{r+g}(V)}{\phi_{r+g}(K)} \tag{2.28}$$

Where

$\frac{\phi_{r+g}(V)}{\phi_{r+g}(K)}$ is the expected present value of one dollar payment when bankruptcy happens.[6] The expression for ϕ can be found in Lemma 2 from Perrakis and Zhong (2013):

$$\phi_r(V) = \begin{cases} V^{\beta+\frac{1}{2}} e^{\frac{\epsilon}{2}} W_{k,m}(x), \beta < 0, r \neq 0 \\ V^{\beta+\frac{1}{2}} e^{\frac{\epsilon}{2}} M_{k,m}(x), \beta > 0, r \neq 0 \end{cases} \tag{2.29}$$

Where

$$x = \frac{|r-q|}{\theta^2 |\beta|} V^{-2\beta}, \epsilon \qquad r-q)\beta), m = \frac{1}{4|\beta|}$$

$$k = \epsilon \left(\frac{1}{2} \quad \frac{1}{4\beta} \right) - \frac{r}{2|(r-q)\beta|}$$

Where $W_{k,m}(x)$ and $M_{k,m}(x)$ are Whittaker functions[7].

Firm value v :

$$v(V,K) = V + TB(V,K) - BC(V,K) \tag{2.30}$$

Where:

$$TB(V,K) = \frac{wC}{r} - \frac{wC}{r} \frac{\phi_r(V)}{\phi_r(K)} \tag{2.31}$$

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[6] The cumulative default probability $A(T)$ and the present value of one dollar payments when default happens $B(T)$ are very crucial for our estimation. Perrakis and Zhong (2013) applied Laplace-transformation and inverse Laplace-transformation to get the closed-form solution for them under finite maturity. The details and proofs of these equations can be found from Proposition 2 in Davydov and Linetsky (2001) and Appendix A in Perrakis and Zhong (2013).

[7] To avoid the enormous time and resources consumption when computing the Whittaker Function in Matlab, we used numerical method instead of the Matlab build-in function to calculate the Whittaker Function. It will be discussed in detail in a later section of this paper.

$$BC(V,K)=\alpha K\frac{\phi_r(V)}{\phi_r(K)} \tag{2.32}$$

Then:

$$v(V,K)=V+\frac{wC}{r}-\frac{wC}{r}\frac{\phi_r(V)}{\phi_r(K)}-\alpha K\frac{\phi_r(V)}{\phi_r(K)} \tag{2.33}$$

Where w is the tax rate. The intuition and structure here is similar with the one in Leland (1994b).

Equity value E :

$$E=v-D \tag{2.34}$$

Then:

$$E=V+\frac{wC}{r}\left[1-\frac{\phi_r(V)}{\phi_r(K)}\right]-\frac{\phi_r(V)}{\phi_r(K)}\alpha K-\frac{C+gP}{r+g}\left(1-\frac{\phi_{r+g}(V)}{\phi_{r+g}(K)}\right)-(1-\alpha)K\frac{\phi_{r+g}(V)}{\phi_{r+g}(K)} \tag{2.35}$$

3. Empirical estimation method and techniques

3.1 Parameters to be estimated

The unlevered firm value is the unobservable basic process in structural models. Our task here is to estimate the components of the unlevered firm value process; for this we need the parameters of the basic unlevered firm value process. In the Leland (1994b) model, the asset dynamics of this unlevered firm value are:

$$\frac{dV}{V}=\mu(V,t)dt+\sigma dz \tag{3.1}$$

Since we are interested in the risk neutral version of this process, the only parameter that needs to be estimated is σ . According to the model specification, the endogenous default boundary should be another important component to be estimated for the Leland (1994b) model. However, for simplicity of the calculation and the direct examination of the effect of time-varying volatility, we set the default boundary exogenously, for both the Leland (1994b) and the Perrakis and Zhong (2013) models. We implemented the KMV default boundary in our estimation.[8] Thus, the only difference between these two models is the form of the diffusion volatility.

The CEV risk neutral asset dynamics for the unlevered firm value in Perrakis and Zhong (2013) is:

$$dV=\left(r\text{-}q\right)Vdt+\theta V^{\beta}dW \tag{3.2}$$

Then for this model, the parameters that need to be estimated are θ and β. β is the elasticity coefficient for the CEV process as mentioned in the previous section. However, θ does not have a direct economic intuition here. Thus I define a new variable σ_0 here, which satisfies the following relationship:

$$\sigma_0=\theta V_0^{\beta} \tag{3.3}$$

Where V_0 is the initial value of firm, and is scaled to value 1 in our estimation.[9] Obviously, σ_0 here is the initial volatility of the firm value. Since it has a straightforward economic intuition, we estimate this parameter instead of θ. There is a good reason we did this. Since we apply numerical methods to estimate the parameters we are interested in, we have to guess an initial value of the parameters to start the process. If the parameters have straightforward economic intuition, it will be easier for us to give those parameters meaningful initial guesses. Moreover, once we get the results of our estimation, it will give us the convenience to check whether the

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8 Moody's KMV defines as trigger value $K = P_{Short} + 0.5 * P_{Long}$. Where P represent firm liability.

9 Other related balance sheet items are also scaled accordingly. The details will be discussed in the data section.

results fall into a reasonable range. Then, after we get the value of σ_0, we can calculate the value of θ by applying the above relationship.

3.2 Moment conditions and the GMM method

To estimate those parameters we are interested in by the GMM method, we first construct our moment conditions which will be used to calculate the objective function for the GMM method. Follow Huang and Zhou (2008), we use the term structure of CDS spreads and volatility as part of our moment conditions. In addition, we incorporate equity value and leverage ratio into our moment condition. In particular, we have nine moment conditions: equity value, equity volatility, leverage ratio, and six different maturity CDS spreads (1, 2, 3, 5, 7, and 10 years). First, we will present the method used to calculate these values.

Model predicted CDS spreads:[10]

$$CDS\left(0,T\right)=\frac{\left(1\text{-}R\right)\sum_{i=1}^{4T}D\left(0,T_i\right)\left[Q\left(0,T_{i\text{-}1}\right)\text{-}Q\left(0,T_i\right)\right]}{\sum_{i=1}^{4T}D\left(0,T_i\right)Q\left(0,T_i\right)/4} \tag{3.4}$$

Here we assume that the CDS premium payment is quarterly. R is the recovery rate, $D\left(0,T_i\right)$ is the discount factor, and $Q\left(0,T_i\right)$ is the survival probability during the time interval $\left[0,T_i\right]$. The CDS spreads can be calculated while the survival probability can be derived based on the asset diffusion process. Actually, the survival probability $Q\left(0,T\right)$ is the crucial connection bridge between our empirical estimation and the model:

$$Q\left(0,T\right)=1\text{-}A\left(0,T\right) \tag{3.5}$$

[10] This formula for CDS spreads is directly adopted from Huang and Zhou (2008).

Where $A(0,T)$ is the first passage default probability. Unlike the first passage default probability of the Leland (1994b) model, the probability for the Perrakis and Zhong (2013) CEV model is very complicated. It has already been proved that the first passage default probability for the CEV model can be calculated by the formula bellow:[11]

$$A(T) = inverselaplace\left(\frac{1}{\lambda} \bullet \frac{\phi_\lambda(V)}{\phi_\lambda(K)}\right) \tag{3.6}$$

Where ϕ can be calculated by applying equation (2.29). However, the build-in functions in Matlab to calculate the Whittaker Functions W and M are very inefficient. Moreover, there is an overflow problem with the Matlab programing,[12] so we used numerical methods to approximate the Whittaker Functions in our estimations. The details can be found in Appendix A.

Then applying the inverse-Laplace transformation we can get the default probabilities we need for the CDS spreads.[13]

Model predicted equity value:

$$\begin{aligned} E &= v - D \\ &= V + \frac{wC}{r}\left[1 - \frac{\phi_r(V)}{\phi_r(K)}\right] - \frac{\phi_r(V)}{\phi_r(K)}\alpha K - \frac{C+gP}{r+g}\left(1 - \frac{\phi_{r+g}(V)}{\phi_{r+g}(K)}\right) - (1-\alpha)K\frac{\phi_{r+g}(V)}{\phi_{r+g}(K)} \end{aligned} \tag{3.7}$$

Notice that in the expression for the equity value, there are several ϕ-values in our calculations. For these we adopt the same numerical method as the one presented in Appendix A to approximate the value of the Whittaker Function and ϕ.[14]

[11] The proof can be found in Perrakis and Zhong (2013) Appendix A and in Proposition 2 in Davydov and Linetsky (2001).

[12] Extremely small numbers in Matlab will be treated as zero. However, when this number takes the position of denominator, overflow will happen.

[13] The numerical method of inverting Laplace transforms can be found in Section 5 of Kuo and Wang (2003). We present the method in Appendix B in our paper. Note that we have modified some of the parameters of their method.

Model predicted equity volatility:

$$\sigma_{\mathrm{E}} = \frac{\frac{\partial E}{\partial V}\theta V^{\beta+1}}{E} \tag{3.8}$$

This relationship between equity volatility and firm value volatility was first pointed out by Merton (1974). We can see that both the equity volatility and firm value volatility here are time-varying. Since we already have the expression for equity value E, the crucial part of this formula now is the partial derivative of equity value with respect to firm value. We can find the answer in Appendix B of Perrakis and Zhong (2013). However, if we directly apply their expressions, we will encounter some technical issues such as the overflow of the Matlab programming. Therefore, we have to find a way to numerically estimate the partial derivative. We provide our numerical solution in Appendix C of this paper.

Model predicted leverage ratio:

$$Lev = \frac{D}{D+E} \tag{3.9}$$

Where D and E can be calculated by equation (2.18) and equation (2.24).

Then we can use these values to build up our moment conditions and conduct our empirical test. In our first exercise, we only incorporate seven moments: equity volatility, and term structure of CDS spreads. We denote the estimation parameter vector $\psi_1 = (\sigma_0, \beta)$. The Leland (1994b) model will be a special case when $\beta = 0$. Thus, we will have the following over-identified system:

[14] The proof can also be found in Appendix B of Perrakis and Zhong (2013)

$$f_1(\psi_1,t)=\begin{Bmatrix} \sigma_E(t)-\sigma_E(t) \\ CDS(t,T_1)-CDS(t,T_1) \\ \cdots\cdots \\ CDS(t,T_i)-CDS(t,T_i) \end{Bmatrix} \tag{3.10}$$

Where f_1 is a function of our parameter set at each time point $t=1,2,...,T$. σ_E, is observed equity volatility. $CDS(t,T_i)$ is observed term structure of CDS spreads. Under the null hypothesis that the model is correct, we have:

$$E\left[f_1(\psi_1,t)\right]=0 \tag{3.11}$$

By applying the GMM method, we want to minimize $E\left[f_1(\psi_1,t)\right]$. We set:

$$G_1(\psi_1,t)=\frac{1}{T}\sum_1^T f_1(\psi_1,t) \tag{3.12}$$

Then we can estimate our parameter ψ_1 by minimizing the following objective function:

$$\psi_1=\arg\min G_1(\psi_1,t)' W G_1(\psi_1,t) \tag{3.13}$$

Where W is the inverse of the variance-covariance matrix of the moment conditions. In Huang and Zhou (2008) and Elkamhi, Ericsson and Jiang (2011), they specify that the weighted matrix W is the asymptotic covariance matrix. However, from page 443-447 in Green's Econometrics Analysis (Sixth Edition), we can see that the most efficient weighted matrix is the inverse of the variance and covariance matrix of the moments. Moreover, intuitively, due to the different magnitude of each moment, if we directly apply the covariance matrix as our weighted matrix, it will definitely give different weights to different moments in our estimation, causing bias. On the other hand, the inverse of the variance-covariance matrix will give each moment equal weight, incorporating information in different moments equally. Therefore, we chose the inverse of variance matrix to be our weighted matrix. We also used the variance matrix as a robustness check and, as we predicted, the results were not as good. The results of this robustness check are not reported in this paper. The details of the GMM method are presented in Appendix D of this paper.

In our next exercise, we add leverage ratio and equity value in our estimation. Thus, we have the following over-identified restrictions:

$$f_2(\psi_2, t) = \begin{Bmatrix} CDS(t, T_1) - CDS(t, T_1) \\ \ldots\ldots \\ CDS(t, T_i) - CDS(t, T_i) \\ \sigma_E(t) - \sigma_E(t) \\ E(t) - E(t) \\ Leverage(t) - Leverage(t) \end{Bmatrix} \tag{3.14}$$

Where ψ_2 is the parameter vector. CDS, σ_E, E, and $Leverage$ are the observed CDS term structure, equity volatility, equity value, and leverage ratio respectively. Then, it is obvious that each moment in this system is the difference between the observed value and its model calculated counterpart. This difference is the pricing error of the model.

As in the previous exercise, the null hypothesis is that the model is specified correctly; we have:

$$E\left[f_2(\psi_2, t)\right] = 0 \tag{3.15}$$

To minimize $E\left[f_2(\psi_2, t)\right]$ by the GMM method, we set:

$$G_2(\psi_2,t)=\frac{1}{T}\sum_{1}^{T} f_2(\psi_2,t) \tag{3.16}$$

Our objective function will be:

$$\psi_2=\arg\min G_2(\psi_2,t)^{'} W G_2(\psi_2,t) \tag{3.17}$$

The calculation of the weighted matrix W is the same as the one in the previous exercise.

4. Data description

We incorporate data from different sources: Credit Default Swap market, equity market, option market, debt market, and firm financial statements.

Our CDS spreads data is from the Markit database. The data period is from January 2001 to December 2011. We restrict our sample to United States firms and the currency is in US dollars. Moreover, we focus on CDS contracts with modified restructuring (MR) policy because they are the most popular in the US market.[15] Observations for which important variables[16] are missing or have unreasonable values[17] are deleted from our sample. After these cleaning up steps, the observations left will constitute our final sample. Since we need to build up our dataset monthly while all the CDS data from Markit is daily, we chose the last Wednesday of each month to represent that month and convert our daily data into monthly data. Eventually, to make our estimation more reliable, we only select those firms which have at least 60 months' consecutive observations.

[15] We follow Huang and Zhou (2008) for this part.

[16] "Important variables" means those variables which will be used in our estimation, such as Recovery Rate, CDS Spreads, etc..

[17] "Unreasonable" means the value of the variable does not make economic sense, for example, negative recovery rates or bigger than one CDS spreads.

The financial statement variables and equity variables are acquired from the Compustat and CRSP databases. After unifying their measurement units[18] and standardizing[19] them, we expanded their frequency form quarterly to monthly by SAS.[20] Equity value is calculated as stock price times shares outstanding, firm value is calculated as book value of debt plus market value of equity, and payout rate is calculated as dividend payments plus interest payment scaled by the firm's total assets. As for the CDS spreads data, we eliminated those observations whose important variables values are missing or unreasonable.

The option implied volatility is obtained from the OptionMetrics database and the realized volatility is obtained from the TAQ database on five-minutes intervals and is then converted to monthly data. In our main test, we used implied volatility only, with the realized volatility used just for robustness checks. The reason is that the option implied volatility can reflect the information from option market. Cao, Yu and Zhong (2010) argue that implied volatility is a more efficient forecast for future realized volatility.

[Insert Table 1]

Following Longstaff and Schwartz (1995) and Collin-Dufresne and Glodstein (2001), we used the term structure of interest rates instead of a constant interest rate as our risk free rate. We interpolated our risk rate term structure from observed 3 month, 6 month Libor rates and 1, 2, 3, 5, 7, 10 years interest rate swap rates.

After merging and cleaning all the data from the different databases by the above criteria, we have 104 firms surviving in our total sample.

[Insert Table 2]

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[18] Unit for Compustat items is $millions and unit for CRSP items is $thousands.

[19] The standardization is achieved by scaling the accounting items to the total assets of the firm in the first observation for each individual firm.

[20] Expand Procedure is used in SAS.

5. Estimation Results and Analysis

In this section, we summarize and analyze the findings of the empirical test of the GMM estimator defined in the last section using the CDS spreads term. On the basis of these results, we also compare the Leland (1994b) and Perrakis and Zhong (2013) models in terms of their goodness of fit. For the estimation, we use two alternative data sets in our tests: the first is the 7-moment set, which includes 1-year, 2-year, 3-year, 5-year, 7-year and 10-year CDS spreads and implied equity volatility; and the second set is the 9-moment, which contains two additional moments, equity value and leverage.

5.1 Summary Statistics

In this paper, we used data from different sources and converted their frequencies to monthly. Table 1 defines the variables used in our empirical tests and their units of measurement. Table 2 lists the 103 firms in our data base and provides descriptive statistics of their most important characteristics. Table 3 provides summary statistics on firm characteristic and CDS spreads of our sample firms across both rating and sector categories in terms of average value. As can be seen from Panel A of this table, our sample firms' debt ranges from triple-C to triple-A. Nonetheless, most of our sample is concentrated in the single-A and triple-B categories, which account for 82% of the total, consistent with the study of Huang and Zhou (2008). In terms of the averages in the entire sample, the 5-year CDS spread is 61.76 basis points, implied equity volatility is 27.31%, and leverage ratio is 37.91%. As we expected, the CDS spreads, volatility and leverage increase as rating decreases. However, we note that single-B and triple-C are two exceptions, with the CDS spreads and implied volatilities actually decreasing as rating decreases. Since they only have 1 observation in each category respectively, we attribute this finding to lack of sufficient sample size. Consistent with our intuition, the CDS spreads for all rating are increasing as maturity increases.

[Insert Table 3]

[Insert Table 4]

Figure 1 plots the time series of the average CDS spreads (5-year CDS spreads from January 2001 to December 2011). As presented in the figure, the average CDS spreads show large variation during the period and have two peaks around late 2002 and late 2008 respectively.

[Insert Figure 1]

5.2 Tests for the Leland model

We use Leland (1994b) as our benchmark model. However, to simplify the calculations at this point, we used the fixed exogenous default boundary instead of the endogenous default boundary setting. Hence, the parameter we care about in Leland model is "Sigma", the volatility of the unlevered firm value. In Tables 5 and 7, the parameter estimation results are shown respectively for 9- and 7-moment estimations under the Leland model. As can be seen in Panel C of Tables 5 and 7, the average values of Sigma are 15.57% and 16.18% respectively. Also shown are the test statistics for each rating and sector categories. The T value shown in the table are the average T values of the estimated parameters for each individual firm. We can clearly see that they are all highly significantly different from zero. However, in both tables we could not find a clear pattern about Sigma across different rating groups and different industrial sectors. The column titled "F Value" reports the optimized value of our object function, equation 3.17. And the column titled "J test" reports the value of the J statistic, $J = T \times F\_value$.

[Insert table 5]

[Insert table 7]

In Table 9, the distributions of Sigma under the Leland model across individual firms are shown , respectively for 7- and 9-moment estimations. Firm “Dow Chem Co” has the largest Sigma values for 7- and 9-moment, 0.2974 and 0.2913 respectively; firm “Raytheon Co” has the smallest Sigma values for 7- and 9-moment, 0.05 and 0.05 respectively. We can see that overall, the Leland model with 7-moment conditions will get a bigger estimation for Sigma. However, even if the T-statistics under 7- and 9-moment are both highly significant, we can still observe a significant difference, 40.33 and 64 on average for 7- and 9-moment respectively. Since the T-statistics are calculated as the quotient of the values of Sigma and the standard errors of the sigma, the standard errors for Sigma under 9-moment conditions must be smaller than their counterparts under 7-moment conditions, especially since the values of Sigma under 9-moment conditions are generally smaller. In fact, in our estimation, most of the firms have a smaller standard error under the 9-moment condition, only two firms (“Merck & Co Inc” and “Wal Mart Stores Inc”) have a smaller standard error under 7-moment conditions. Therefore, from the aspect of parameter estimation, the Leland model under 9-moment conditions perform a little bit better than the Leland model under 7-moment conditions due to its smaller estimation standard error. On the other hand, if we take a look at Figure 6, we can see that from the aspect of historical data fitting, both Leland models with 7- and 9-moments are very similar with each other. The 7-moment condition fit is slightly better than the 9-moment condition.

[Insert table 9]

Since Single-A and Triple-B CDS account for 82% in our total sample, their Sigma distribution are shown separately in Table 10. The average Sigma value for A- rated firms with 7- and 9-momnet are 0.1743 and 0.1675 respectively; The average Sigma value for BBB rated firms with 7- and 9-moments are 0.1494 and 0.1444 respectively; The total average Sigma values for these firms with 7- and 9-moments are 0.1607 and 0.1549 respectively. For A rated firms, “Baker Hughes Inc” has the largest Sigma value of 0.2756 and 0.2749 for 7- and 9-moments respectively;

“Raytheon Co” has the smallest Sigma value of 0.05 and 0.05 for 7- and 9-moments respectively. For BBB rated firms, “Dow Chem Co” has the largest Sigma value of 0.2974 and 0.2913 for 7- and 9-moments respectively; “Textron Inc” has the smallest Sigma value of 0.0685 and 0.0507 for 7- and 9-moments respectively. We can clearly see from Table 10 that the Leland model with 7-moment conditions will yield a bigger estimate of sigma than the Leland model with 9-moment conditions, consistent with the result of Table 9.

[Insert Table 10]

5.3 Tests for the CEV model

In the CEV model, we estimate two parameters, Sigma and Beta. Because of the computational complexity mentioned in the previous section of this paper, we estimated positive betas and negative betas separately. Then, we use the F value as the standard to decide whether our firm is positive beta or negative beta (whichever has the smaller F value). Table 6 shows the estimation results for the CEV model using 9 moments. The left panel shows the result for all firms, the middle part shows the firms with positive beta, and the right panel shows the firms with negative beta. We can clearly see that all betas in the left panel are not significant, while most of the betas in the middle panel and the right panel are highly significant. Our total sample here is 103 firms (Republic Services Inc. cannot be calculated for either positive or negative betas), the number of firms with positive betas is 52, and the number of firms with negative betas is 51. Hence, in the entire sample the values of positive betas and negative betas cancel out and make the test statistic not significant. Once we separate the sample into positive and negative betas, they are both very significant. In terms of the averages in each sub-sample, the value of beta is 0.6392 for the positive group and -0.5204 for the negative group, both significant. Looking at the T values, we find that on the individual firm level, all our estimated parameters are significant except for the triple-A firm with positive beta, which has a T value equal to 1.7.

[Insert table 6]

Table 8 shows the estimation results for the CEV model using 7 moments. In contrast with Table 6, the number of firms with negative beta dominates this time. The total sample number is still 103 (Diamond Offshore Drilling Inc. cannot be calculated for both positive and negative betas), but the number of firms with positive beta is only 29, while the number of firms with negative beta is now 74, accounting for 71.84% of the total sample. All betas in the negative group are highly significant. When we directly compare these two CEV models with different moments settings, we find that the CEV model with 7-moments performs better than the CEV model with 9 moments, with a total average J test value equal to 13.51, which is smaller than the J test value in the 9-moment model of 14.39. This finding is confirmed further on, by the figures presented in the next subsection. In addition, all T values in this test are highly significant. We can also see that in Table 8, the T values for both positive and negative groups are much larger than the corresponding T values in Table 6. This finding implies that from the point of view of parameter estimation, the CEV model with 7-moment conditions also perform better than the CEV model with 9-moment conditions.

[Insert table 8]

In Tables 11 and 12, we show the beta distributions across our sample firms. Table 11 presents the beta distribution for the CEV model with 9-moment conditions. There are 51 firms with negative betas and 52 firms with positive betas. Panel A shows the firms with negative betas. Firm “Gen Mls Inc” has the smallest beta. The average beta, average J statistic, and average T value are -0.5204, 14.159 and 64.8393 respectively for the negative group ; panel B shows the firms with positive betas. Firm “AmerisourceBergen Corp” has the largest beta. The average beta, average J statistic, and average T value are 0.6392, 14.6222 and 5.656 respectively for the positive group. Table 12 presents the beta distribution for the CEV model with 7-moment conditions. There are 74 firms with negative betas and 29 firms with positive betas. Panel A shows the firms with negative betas. Firm "Smithfield Foods Inc" has the smallest beta. The average beta, average J statistic, and average T value are-0.9508, 13.4377 and 200.7618 respectively for the negative group; panel B shows the firms with positive betas. Firm "Gen Mls Inc" has the largest beta. The average beta, average J statistic, and average T value are 0.5201, 13.7058 and 61.777 respectively for the positive group. We can see that, in both 7- and 9-moment condition CEV model, the negative beta groups have larger T values and smaller J statistics. This implies that for both historical data fitting and firm level parameter estimation levels, the negative beta group performs better than the positive beta group. On the other hand, we can directly compare the test statistics between 7- and 9-moment condition models. Clearly, in both positive and negative beta groups, the 7-moment condition CEV model has a much larger T value and a smaller J statistic, implying that the leverage ratio as a moment condition in the CEV model does not improve the fitting and estimation ability of the model.

[Insert table 11]

[Insert table 12]

5.4 Comparing results

In Figure 2 and Figure 4, we show the 5-year CDS spreads, the historical data and the fitted values of both models using 7-moment and 9-moment estimations respectively. The solid line represent the observed historical data, the dot line represent the CEV model calculated 5-year CDS spreads, and the dashed line represent the Leland model-calculated 5-year CDS spreads. Consistent with Huang and Huang (2013), the "Credit Spread Puzzle" can be observed in the figures. The Leland model calculated 5-year CDS spreads are consistently underestimating the observed spreads during the whole period. . On the other hand, we can see that the CEV model calculated CDS spreads are much higher, and follow the trend more precisely than the Leland model. At this point, we can state with confidence that the introduction of time varying volatility can enhance the fitting ability of the structural model significantly.

[Insert figure 2]

[Insert figure 4]

However, when we compare the two CEV models with different moments settings, we find that the CEV model with 7 moments outperforms the CEV model with 9 moments. In Figure 6, we put all model and moments combinations together, and it is clear that the CEV model with 7 moments is the best.

[Insert figure 6]

Figures 3 and 5 present the historical data fitting of equity volatility for the Lelandand CEV models with 7- and 9-moment conditions respectively. The Leland- calculated equity volatility is a little more volatile than the CEV- calculated equity volatility. However, there is no other obvious difference between the Leland and CEV models with different moment conditions. We attribute this to the weight of equity volatility as a moment condition in the estimation, which is not as large as the weight of CDS spreads since there are five CDS spreads moment conditions and only one equity volatility moment condition.

We speculate that making Leverage a moment condition would decrease the model- calculated CDS spreads. In Perrakis and Zhong (2013), the authors have similar findings in their model calibration.

At this point, we state that the introducing of time-varying volatility can improve the historical data fitting for structural model significantly, and the 7-moment condition has a better performance compared to the 9-moment condition. However, a comprehensive comparison between the CEV and Leland models should be done in a more systematic way, such as by comparisons of the mean square errors of the moment conditions across all the firms. In addition, we can differentiate the sample firms into different quartiles by beta values and investigate the effect of firm characteristics on beta value by regression. Last but not least, the effect of leverage as a moment condition should be examined thoroughly, ,since the major difference between 7- and 9- moment conditions is the leverage ratio.

6. Conclusion and suggestions for future research

In trying to explain the "Credit Spread Puzzle", we empirically examined two competing structural models: Leland (1994b) and Perrakis and Zhong (2013) Constant Elasticity of Variance (CEV) model. The sample we applied in our test covered the time period from 2001 to 2011. The GMM method was used in this paper to conduct the parameter estimation.

One of our most important findings and conclusions is that the introduction of time-varying volatility into the structural model can significantly improve the model fitting compared to the constant volatility one. We found that most of the betas in the CEV model are highly significant and the time series figures of model calculated CDS spreads show that the Perrakis and Zhong (2013) model performs much better than Leland (1994b) and can fit the historical CDS spreads data better.

Another finding is that Leverage as a moment condition in the GMM test has the effect of driving the model predicted CDS spreads downwards, while CDS spreads as moment conditions have the opposite effect. This finding is consistent with the finding in Perrakis and Zhong (2013) in their calibration.

Last, we note several ideas for expanding and solidifying the conclusions of this paper. First, we note that in our estimation we only used option implied volatility to do the calculations. As a robustness check, the realized volatility can be used to conduct the same estimations. Second and most important, out of sample test of the two models should definitely be carried out to verify the predictive power of the models. Third, since we have already found that leverage as a moment condition has the effect of suppressing the CDS spreads, we should delete leverage and do an 8-moment condition estimation to check independently the effects of equity value on the CDS estimates. Fourth, we should examine the distribution of the CEV model's beta estimates across firms, by identifying firm characteristics that affect their beta values. Last but not least, the difference of the pricing errors between the Leland and CEV models should be examined and quantified more systematically.

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Appendix A: Numerical approximation for Whittaker Functions

The definition of Whittaker Functions can be found on "Wolfram MathWorld" website[21].According to their definition, Whittaker Functions can be written as:

$$\begin{aligned} M_{k,m}(x) &= e^{-x/2} x^{m+1/2} {}_1F_1\left(\frac{1}{2}+m-k, 1+2m; x\right) \\ W_{k,m}(x) &= e^{-x/2} x^{m+1/2} U\left(\frac{1}{2}+m-k, 1+2m; x\right) \end{aligned} \tag{A.1}$$

Where ${}_1F_1$ is the first kind Confluent Hypergeometric Function, and U is the second kind of Confluent Hypergeometric Function[22]:

$${}_1F_1(a;b;x) = 1 + \frac{a}{b}x + \frac{a(a+1)}{b(b+1)}\frac{x^2}{2!} + \ldots = \sum_{k=0}^{\infty} \frac{(a)_k}{(b)_k}\frac{x^k}{k!} \tag{A.2}$$

$$U(a;b;x) = \frac{1}{\Gamma(a)} \int_0^{\infty} e^{-xt} t^{a-1} (1+t)^{(b-a-1)} dt \tag{A.3}$$

Where:

$$a = \frac{1}{2} + m - k; \; b = 1 + 2m$$

$(a)_k$ and $(b)_k$ are rising factorial

The expressions for m and k are given in equation (2.14). For Whittaker M function, the expression is basically an infinite summation. To get reasonable accuracy without sacrificing too much efficiency, we chose to pick out the first 2000 terms in this summation to calculate the function.

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21 Website address: http://mathworld.wolfram.com/WhittakerFunction.html

22 We found that the definitions or expressions for Whittaker Function W and M are not unique. However, what we report here are the most efficient expressions for those two functions to do our task.

On the other hand, however, the estimation for the Whittaker W function is a little bit tricky. The value of parameter "a" for $U\left(a;b;x\right)$ is negative due to its expression. The negative parameter in the Gamma Function will cause a lot of trouble in the calculations: first, when the parameter is a negative integer, the value of the Gamma Function will be infinite; second, for some negative values of the parameter, the magnitude of the value of the Gamma Function will be too large and would cause the overflow of the programming. Therefore, we have to find a way to get rid of the Gamma Function $\Gamma\left(a\right)$ in $U\left(a;b;x\right)$. Fortunately, in our estimation, the calculation of ϕ always comes in pairs under the form of:

$$\frac{\phi_\lambda\left(V\right)}{\phi_\lambda\left(K\right)}$$

Then, we derived the formula for this expression:

$$\begin{aligned}
\frac{\phi_\lambda\left(V\right)}{\phi_\lambda\left(K\right)} &= \frac{V^{\beta+\frac{1}{2}}e^{\frac{\epsilon}{2}V}W_{k,m}\left(x_V\right)}{K^{\beta+\frac{1}{2}}e^{\frac{\epsilon}{2}K}W_{k,m}\left(x_K\right)} \\
&= \frac{V^{\beta+\frac{1}{2}}e^{\frac{\epsilon}{2}V}\, e^{-x_V/2}x_V^{\,m+1/2}\,\frac{1}{\Gamma\left(a\right)}\int_0^\infty e^{-xt}t^{a-1}\left(1+t\right)^{(b-a-1)}\,dt}{K^{\beta+\frac{1}{2}}e^{\frac{\epsilon}{2}K}\, e^{-x_K/2}x_K^{\,m+1/2}\,\frac{1}{\Gamma\left(a\right)}\int_0^\infty e^{-xt}t^{a-1}\left(1+t\right)^{(b-a-1)}\,dt} \\
&= \frac{V^{\beta+\frac{1}{2}}e^{\frac{\epsilon}{2}V}\, e^{-x_V/2}x_V^{\,m+1/2}\int_0^\infty e^{\left(-x_Vt+(a-1)\ln(t)+(b-a-1)\ln(1+t)\right)}dt}{K^{\beta+\frac{1}{2}}e^{\frac{\epsilon}{2}K}\, e^{-x_K/2}x_K^{\,m+1/2}\int_0^\infty e^{\left(-x_Kt+(a-1)\ln(t)+(b-a-1)\ln(1+t)\right)}dt} \\
&= \frac{V^{\beta+\frac{1}{2}}e^{\frac{\epsilon}{2}V}\, e^{-x_V/2}x_V^{\,m+1/2}\int_0^\infty e^{\left(-x_Vt+(m-k-0.5)\ln(t)+(m+k-0.5)\ln(1+t)\right)}dt}{K^{\beta+\frac{1}{2}}e^{\frac{\epsilon}{2}K}\, e^{-x_K/2}x_K^{\,m+1/2}\int_0^\infty e^{\left(-x_Kt+(m-k-0.5)\ln(t)+(m+k-0.5)\ln(1+t)\right)}dt}
\end{aligned} \tag{A.4}$$

After eliminating the Gamma Function $\Gamma(a)$, the calculation became fairly easy and straightforward. In addition, we did some transformations to the integrand of the above expression. The reason is that we have to eliminate the parameter a on the exponential due to its magnitude, which might cause some calculation problems. We can clearly see that after the transformation, the positive a and negative a will offset each other to some extent.

Even though we use the definition of Whittaker functions to approximate the calculation of the exact function,.,the results we get from these approximations are very accurate when compared to the results we get from the built-in function of Matlab and Mathmatica.

Appendix B: Numerical method of inversion of Laplace transformation.

We directly apply the method provided in Kuo and Wang (2003) to do the Inverse-Laplace transformation. The method is presented as follows:

$$f_n^* = \sum_{k=1}^{n} w(k,n) f_{k+B}(t) \tag{B.1}$$

Where f_n^* is the inverse-Laplace transformation. Breaking down this expression, we have:

$$w(k,n) = (-1)^{n-k} \frac{k^n}{k!(n-k)!} \tag{B.2}$$

Which is the extrapolation weight for an n-point Richardson extrapolation. $f_{k+B}(t)$ is given by:

$$f_n(t) = \frac{\ln(2)}{t} \frac{2n!}{n!(n-1)!} \sum_{k=0}^{n} (-1)^k \binom{n}{k} f\left((n+k)\frac{\ln(2)}{t}\right) \tag{B.3}$$

Where f is the Laplace transformation:

$$f(\lambda)=\int_0^\infty e^{-\lambda t} f(t)\,dt$$

Therefore, the counterpart of this Laplace transformation in our paper is $\frac{1}{\lambda}\bullet\frac{\phi_\lambda(V)}{\phi_\lambda(K)}$. Substituting this expression into the formula presented above, we can get the inverse-Laplace transformation for our calculation.

However, we still made some small modifications to this method. In Kuo and Wang (2003), the authors stated that the typical value for parameter B is 2 or 3, and the typical range for parameter n is from 5 to 10. In our work, for β>0, we found that n=8 and B=2 is the most efficient value. For β<0, we found that for large values of n, the algorithm did not converge very well, especially for very short time periods. Therefore, for time periods less than two quarters, we set n=4, otherwise, n=5, and we set B=0 for all time periods.

Appendix C: The numerical approximation for the partial derivative of equity value with respect to firm value.

In Appendix B of Perrakis and Zhong (2013), the authors gave their expression for the partial derivative of equity value with respect to firm value as follows:

$$\frac{\partial E(V,K)}{\partial V}=1-\left[\frac{wC}{r}+\alpha K\right]\left[\frac{1}{\phi_r(K)}\frac{\partial\phi_r(V)}{\partial V}\right]+\left[\frac{C+gP}{r+g}-(1-\alpha)K\right]\frac{1}{\phi_{r+g}(K)}\frac{\partial\phi_{r+g}(V)}{\partial V} \quad \text{(C.1)}$$

Where:

$$\frac{\partial\phi_r(V)}{\partial V}=\begin{cases}\left[\beta+0.5+0.5Vx'\varepsilon\right]V^{\beta-0.5}e^{0.5\varepsilon x}W_{k,m}(x)+V^{\beta+0.5}e^{0.5\varepsilon x}W'x', if\,\beta<0\\ \left[\beta+0.5+0.5Vx'\varepsilon\right]V^{\beta-0.5}e^{0.5\varepsilon x}M_{k,m}(x)+V^{\beta+0.5}e^{0.5\varepsilon x}M'x', if\,\beta>0\end{cases}$$

Where:

$$W^{'} = -\left(\frac{k}{x} - \frac{1}{2}\right) W_{k,m}\left(x\right) - \frac{W_{k+1,m}\left(x\right)}{x}$$

$$M^{'} = \frac{M_{k+1,m}\left(x\right)(k+m+0.5)}{x} - (\frac{k}{x} - \frac{1}{2})M_{k,m}\left(x\right)$$

However, when we apply this expression to our empirical work, the programming cannot generate the right value. After scrutinizing the code piece by piece, we located the problem in the part of the partial derivative of ϕ with respect to firm value. Therefore, we implemented the following definition of the derivative to get around this problem:

$$\frac{dy}{dx} = \frac{f\left(y + \Delta \qquad \Delta\right.}{2\Delta}$$

Hence:

$$\begin{aligned} \frac{\partial \phi_r\left(V\right)}{\partial V} &= \frac{\phi_r\left(V + \Delta \qquad \Delta\right.}{2\Delta} \\ \frac{\partial \phi_{r+g}\left(V\right)}{\partial V} &= \frac{\phi_{r+g}\left(V + \Delta \qquad \Delta\right.}{2\Delta} \end{aligned} \tag{C.2}$$

Incorporating the adjacent multiplier, we got:

$$\begin{aligned} \frac{1}{\phi_r\left(K\right)} \frac{\partial \phi_r\left(V\right)}{\partial V} &= \frac{\phi_r\left(V + \Delta \qquad \Delta\right.}{2\Delta \qquad \Delta} \\ \frac{1}{\phi_{r+g}\left(K\right)} \frac{\partial \phi_{r+g}\left(V\right)}{\partial V} &= \frac{\phi_{r+g}\left(V + \Delta \qquad \Delta\right.}{2\Delta \qquad \Delta} \end{aligned} \tag{C.3}$$

We can see that, after this transformation, the expression changes into a familiar ϕ over ϕ shape. Therefore, we can apply the technique derived in Appendix A to approximate its value.

Substituting into the whole expression of the partial derivative of equity value with respect to firm value, we can get:

$$\frac{\partial E(V,K)}{\partial V} = 1 - \left[\frac{wC}{r} + \alpha K\right]\left[\frac{\phi_r\left(V + \triangle \quad \triangle\right.}{2\triangle \quad \triangle}\right] . \qquad (C.4)$$
$$\left[\frac{C+gP}{r+g} - (1-\alpha)K\right]\left[\frac{\phi_{r+g}\left(V + \triangle \quad \triangle\right.}{2\triangle \quad \triangle}\right]$$

Appendix D: GMM estimation.

One advantage of the GMM method is that the weight matrix W can be updated during every iteration of the estimation. Therefore, the objective function is evolving during the whole estimation process by incorporating new information from the last iteration. The entire estimation starts from setting the weight matrix W equal to a same dimension identityl matrix I . Then, during the first iteration, the objective function will be like:

$$\psi = \arg\min G(\psi_1, t)^{'} G(\psi_1, t) \qquad (D.1)$$

Which is similar to an OLS estimation. After we get the parameter vector ψ , we can build up the moment conditions and weight matrix W_1 based on this parameter vector. Then the objective function is updated to:

$$\psi_2 = \arg\min G(\psi_2, t)^{'} W_1 G(\psi_2, t) \qquad (D.2)$$

Then, the new parameter vector ψ_2 is used to construct a new weight matrix. This procedure will continue for several iterations and eventually converge to the desired parameter. In our estimation, we set our number of iterations to 8.

Table 1: Description of the important variables

This table shows how we constructed the most important variables that will be heavily used in our estimation. Panel A shows the variables from COMPUSTAT database; panel B shows the variables from CRSP; panel C shows the data from Markit database; panel D shows volatility measurements.

Panel A: Description of Compustat variables

Variable	Description
LCTQ	Current liability total. It is used to calculate the value of the asset as following: V=LCTQ+LLTQ+PRC*SHROUT
LLTQ	Long term liability total. It is used to calculate the value of asset.
XINTQ	Interests and related expense. It is used to calculate the payout rate: Payout=XINTQ+DVY
DVY	Cash dividend. It is used to calculate the payout rate.

Panel B: Description of CRSP variables

Variable	Description
PRC	Stock price. It is used to calculate the value of asset. See panel A.
SHROUT	Shares Outstanding. It is used to calculate the value of asset. See panel A.

Panel C: Description of Markit variables

Variable	Description
spre1y	CDS spread for 1 year maturity
Spre2y	CDS spread for 2 year maturity
Spre3y	CDS spread for 3 year maturity
Spre5y	CDS spread for 5 year maturity
Spre7y	CDS spread for 7 year maturity
Spre10y	CDS spread for 10 year maturity
recovery_new	Recovery Rate

Panel D: Volatility measurement

Variable	Description
Option implied volatility	Extracted from OptionMetrics database.
Realized volatility	Extracted from TAQ database by five minutes interval, and converted to monthly frequency.

Table 2: Summary Statistics of Individual Firms

This table reports the summary statistics of individual firms. We have 104 firms in our total sample. Note that in sector column, *BM*, *CG*, *CS*, *EN*, *HC*, *IN*, *TE*, *TS* are abbreviations of *Basic Materials*, *Consumer Goods*, *Consumer Services*, *Energy*, *Healthcare*, *Industrials*, *Technology* and *Telecommunications Services*, respectively. The *payout ratio* is the sum of cash dividend and interest expense divided by the total asset. The *recovery rates* are the estimated recovery rates reported in Markit datasets. The *implied volatilities* are extracted from *Optionmetrics* for the at the money call options.

Company Name	Sector	Rating	Begin date	End date	Total Asset (billion)	Payout Ratio	Leverage Ratio	Recovery Rate	Implied volatility
3M Co	IN	AA	04/2003	12/2011	69.56	0.01	0.18	0.40	0.22
Abbott Labs	HC	AA	10/2003	12/2011	98.95	0.01	0.23	0.40	0.21
Air Prods & Chems Inc	BM	A	04/2003	09/2008	20.78	0.01	0.30	0.40	0.22
Alcoa Inc.	BM	BBB	08/2001	09/2008	45.79	0.01	0.42	0.40	0.34
AmerisourceBergen Corp	CS	BBB	02/2004	12/2011	17.37	0.00	0.55	0.40	0.27
Anadarko Pete Corp	EN	BBB	01/2003	09/2008	40.52	0.01	0.48	0.40	0.31
Anheuser Busch Cos Inc	CG	A	06/2003	10/2008	51.64	0.01	0.25	0.40	0.18
APACHE CORP	EN	A	03/2003	09/2008	31.58	0.00	0.30	0.40	0.31
Archer Daniels Midland	CG	A	06/2003	09/2008	31.83	0.01	0.44	0.40	0.30
Arrow Electrs Inc	CG	BBB	11/2001	12/2011	7.43	0.00	0.57	0.40	0.38
Autozone Inc	CS	BBB	03/2003	07/2011	12.58	0.00	0.35	0.40	0.27
Avon Prods Inc	CG	BBB	01/2003	12/2011	19.04	0.01	0.25	0.40	0.31
Baker Hughes Inc	EN	A	11/2001	09/2008	20.82	0.01	0.17	0.40	0.34
Baxter Intl Inc	HC	A	02/2002	12/2011	36.91	0.01	0.26	0.40	0.26
Black & Decker Corp	CG	BBB	05/2002	01/2010	8.50	0.01	0.47	0.41	0.32
Boeing Co	IN	A	04/2001	09/2008	92.46	0.01	0.50	0.40	0.28
BorgWarner Inc	CG	BBB	11/2001	09/2008	5.21	0.01	0.43	0.40	0.31

Bristol Myers Squibb Co	HC	A	04/2003	12/2011	64.12	0.02	0.26	0.40	0.25
Campbell Soup Co	CG	A	06/2002	10/2011	17.43	0.01	0.32	0.40	0.21
Caterpillar Inc	IN	A	04/2001	09/2008	67.27	0.01	0.54	0.40	0.28
CenturyTel Inc	TS	BBB	03/2003	04/2008	8.99	0.01	0.50	0.40	0.22
Clorox Co	CG	BBB	07/2004	07/2009	13.16	0.01	0.32	0.40	0.21
Coca Cola Entpers Inc	CG	A	06/2003	09/2008	30.04	0.01	0.66	0.40	0.23
Colgate Palmolive Co	CG	AA	08/2003	12/2011	41.92	0.01	0.19	0.40	0.20
ConAgra Foods Inc	CG	BBB	08/2001	07/2011	20.08	0.03	0.42	0.40	0.22
ConocoPhillips	EN	A	01/2003	09/2008	152.40	0.01	0.45	0.39	0.25
Costco Whsl Corp	CS	A	07/2004	07/2011	35.93	0.00	0.29	0.40	0.25
CSX Corp	IN	BBB	01/2003	09/2008	29.17	0.01	0.58	0.40	0.29
Cytec Inds Inc	BM	BBB	02/2004	12/2011	4.30	0.00	0.49	0.40	0.36
Danaher Corp	IN	A	01/2004	12/2011	29.13	0.00	0.23	0.40	0.25
Diamond Offshore Drilling	EN	A	07/2003	09/2008	10.89	0.02	0.19	0.40	0.37
Dover Corp	IN	A	12/2004	12/2011	12.74	0.01	0.31	0.40	0.29
Dow Chem Co	BM	BBB	01/2002	09/2008	65.87	0.02	0.44	0.40	0.28
Eastman Chem Co	BM	BBB	01/2003	09/2008	8.57	0.01	0.52	0.40	0.25
FedEx Corp	IN	BBB	08/2002	07/2011	36.34	0.00	0.30	0.40	0.28
Gen Dynamics Corp	IN	A	11/2004	12/2011	41.54	0.01	0.36	0.40	0.24
Gen Mls Inc	CG	BBB	04/2002	07/2011	31.57	0.02	0.39	0.40	0.19
Goodrich Corp	IN	BBB	09/2001	09/2008	9.19	0.01	0.52	0.40	0.33
Halliburton Co	EN	A	02/2003	09/2008	34.94	0.01	0.30	0.40	0.33
H J HEINZ CO	CG	BBB	04/2001	10/2011	21.96	0.02	0.38	0.41	0.21
Home Depot Inc	CS	A	02/2002	09/2008	95.07	0.01	0.22	0.41	0.28
Honeywell Intl Inc	IN	A	11/2001	12/2011	54.86	0.01	0.41	0.40	0.30
Intl Business Machs Corp	TE	AA	04/2001	12/2011	233.09	0.01	0.34	0.40	0.25
Intl Paper Co	BM	BBB	04/2001	09/2008	39.13	0.01	0.56	0.40	0.27
Johnson & Johnson	HC	AAA	03/2003	12/2011	207.15	0.01	0.15	0.40	0.17

Kellogg Co	CG	BBB	03/2003	12/2011	27.45	0.01	0.33	0.40	0.18
Kimberly Clark Corp	CG	A	02/2004	12/2011	38.91	0.02	0.29	0.40	0.18
The Kroger Co.	CS	BBB	08/2006	10/2011	33.61	0.01	0.53	0.40	0.29
Eli Lilly & Co	HC	A	06/2003	12/2011	70.15	0.02	0.22	0.40	0.24
Ltd Brands Inc	CS	BB	03/2003	09/2008	12.76	0.01	0.29	0.40	0.32
Lockheed Martin Corp	IN	A	04/2001	12/2011	52.81	0.01	0.45	0.40	0.26
Lowes Cos Inc	CS	A	01/2003	09/2008	54.34	0.00	0.22	0.40	0.28
Marriott Intl Inc	CS	BBB	05/2002	09/2008	18.00	0.00	0.31	0.40	0.30
Masco Corp	CG	BB	07/2002	09/2008	18.45	0.01	0.39	0.41	0.31
Medtronic Inc	HC	A	09/2003	10/2011	62.35	0.01	0.16	0.40	0.24
Merck & Co Inc	HC	AA	03/2004	10/2009	105.48	0.02	0.23	0.40	0.27
Mohawk Inds Inc	CG	BBB	12/2004	12/2011	7.91	0.00	0.44	0.40	0.38
Molson Coors Brewing	CG	BBB	10/2005	12/2011	12.93	0.01	0.41	0.40	0.27
Monsanto Co	BM	A	04/2003	09/2008	31.34	0.01	0.22	0.40	0.32
Motorola Inc	TE	BBB	08/2002	09/2008	57.09	0.01	0.35	0.39	0.38
Newell Rubbermaid Inc	CG	BBB	05/2001	02/2009	11.75	0.02	0.43	0.41	0.30
Nordstrom Inc	CS	A	11/2001	09/2008	10.30	0.01	0.35	0.41	0.37
Norfolk Sthn Corp	IN	BBB	04/2001	09/2008	29.32	0.01	0.54	0.39	0.32
Northrop Grumman Corp	IN	BBB	04/2003	03/2011	36.98	0.01	0.46	0.40	0.22
OCCIDENTAL PETRO	EN	A	09/2002	09/2008	45.32	0.01	0.29	0.40	0.29
Omnicare Inc	CS	BB	11/2004	02/2011	7.65	0.01	0.50	0.26	0.40
Omnicom Gp Inc	CS	BBB	05/2002	12/2011	25.77	0.01	0.47	0.40	0.29
ONEOK Partners LP	EN	BBB	05/2006	12/2011	7.79	0.04	0.53	0.40	0.22
J C Penney Co Inc	CS	BB	06/2001	09/2008	20.37	0.01	0.51	0.38	0.39
Pepsico Inc	CG	A	06/2004	12/2011	124.03	0.01	0.19	0.40	0.18
Pfizer Inc	HC	AA	10/2003	12/2011	234.37	0.02	0.28	0.40	0.24
Pitney Bowes Inc	TE	BBB	11/2003	12/2011	15.76	0.02	0.53	0.40	0.24
PPG Inds Inc	BM	BBB	07/2001	12/2011	17.96	0.01	0.42	0.40	0.27
Praxair Inc	BM	A	10/2003	09/2008	25.16	0.01	0.26	0.40	0.23

Pride Intl Inc	EN	BBB	06/2003	09/2008	6.49	0.00	0.35	0.40	0.38
Procter & Gamble Co	CG	AA	04/2001	12/2011	213.09	0.01	0.25	0.40	0.19
Quest Diagnostics Inc	HC	BBB	09/2005	12/2011	14.15	0.01	0.30	0.40	0.24
Raytheon Co	IN	A	06/2003	12/2011	32.39	0.01	0.42	0.40	0.22
Rep Svcs Inc	IN	BBB	09/2004	12/2011	14.30	0.01	0.43	0.40	0.27
Reynolds Amern Inc	CG	BBB	11/2004	12/2011	26.53	0.02	0.39	0.40	0.23
Rohm & Haas Co	BM	BBB	05/2001	11/2008	15.81	0.01	0.39	0.41	0.27
Ryder Sys Inc	IN	BBB	01/2003	09/2008	7.26	0.01	0.62	0.39	0.29
Safeway Inc	CS	BBB	07/2005	12/2011	20.99	0.01	0.50	0.40	0.31
Schering Plough Corp	HC	A	04/2003	09/2008	40.65	0.01	0.23	0.40	0.28
Sealed Air Corp US	IN	B	02/2006	12/2011	7.05	0.01	0.47	0.40	0.31
Sherwin Williams Co	CG	A	06/2002	12/2011	9.60	0.01	0.31	0.40	0.29
Smithfield Foods Inc	CG	BB	07/2003	08/2008	7.53	0.01	0.57	0.39	0.29
Southwest Airls Co	IN	BBB	06/2003	12/2011	18.62	0.00	0.45	0.39	0.35
Sunoco Inc	EN	BB	07/2003	09/2008	14.55	0.01	0.51	0.40	0.34
SUPERVALU INC	CS	CCC	03/2003	09/2008	15.13	0.01	0.60	0.40	0.28
Sysco Corp	CS	A	03/2005	12/2011	24.48	0.01	0.26	0.40	0.23
Target Corp	CS	A	04/2002	09/2008	64.06	0.00	0.35	0.40	0.30
Textron Inc	IN	BBB	10/2002	09/2008	23.69	0.01	0.58	0.39	0.28
Un Pac Corp	IN	BBB	09/2003	09/2008	45.93	0.01	0.49	0.39	0.24
Utd Parcel Svc Inc	IN	AA	08/2004	12/2011	68.52	0.02	0.32	0.40	0.23
Utd Tech Corp	IN	A	06/2003	09/2008	85.24	0.01	0.33	0.40	0.20
Unvl Health Svcs Inc	HC	BB	03/2004	12/2011	5.10	0.01	0.42	0.40	0.31
UST Inc.	CG	BBB	04/2003	10/2008	8.99	0.03	0.17	0.40	0.22
V F Corp	CG	A	09/2004	12/2011	11.07	0.01	0.26	0.40	0.28
Wal Mart Stores Inc	CS	AA	01/2001	10/2011	296.93	0.01	0.28	0.40	0.23
Waste Mgmt Inc	IN	BBB	01/2004	08/2009	31.50	0.01	0.46	0.40	0.24
Whirlpool Corp	CG	BBB	04/2001	09/2008	12.73	0.01	0.59	0.40	0.33
Wyeth	HC	A	02/2003	07/2009	81.20	0.01	0.28	0.40	0.26

Table 3: Distribution of moments for individual firms under 9-moments models

This table reports the industry and rating distribution of our sample firms in Panels A and B, respectively. “N” represents the number of firms in each category. The 1, 3, 5, 7, 10 years credit default swap spreads are reported as basis points (*bps*). The reported values of all variables are mean values.

Rating/Industry	N	1year spread	2year spread	3year spread	5year spread	7year spread	10year spread	Implied volatility	Equity value	Leverage
Panel A: Rating distribution										
AAA	1	11.51	14.05	16.57	22.33	26.19	30.76	0.1743	0.9223	0.1547
AA	8	15.60	19.39	23.09	30.72	35.54	41.26	0.2262	0.8675	0.2478
A	39	17.30	21.85	26.42	35.80	41.81	48.93	0.2643	1.0128	0.3123
BBB	47	37.94	46.28	54.70	71.91	80.68	90.29	0.2797	0.6947	0.4420
BB	7	83.09	104.23	121.94	154.16	166.00	177.62	0.3377	0.7335	0.4571
B	1	72.47	91.65	111.10	150.17	163.88	176.62	0.3115	0.4640	0.4728
CCC	1	54.54	71.96	89.48	124.22	140.64	157.33	0.2807	0.8826	0.6001
Panel B: Industry distribution										
Basic Materials	10	26.85	33.17	39.11	51.92	59.30	68.51	0.2807	0.9606	0.4022
Consumer Goods	27	32.96	40.62	48.07	62.81	70.40	78.63	0.2499	0.7797	0.3661
Consumer Services	17	45.53	56.66	67.02	87.08	95.90	105.87	0.3000	0.7545	0.3937
Energy	10	30.21	37.82	44.73	58.86	66.96	76.00	0.3134	1.2839	0.3576
Healthcare	12	20.89	26.64	32.54	44.76	51.22	58.01	0.2486	0.7867	0.2516
Industrials	24	29.49	35.94	42.60	56.27	63.71	71.82	0.2740	0.7536	0.4388
Technology	3	37.66	45.74	54.12	68.55	76.46	85.50	0.2869	0.6178	0.4077
Telecommunications Services	1	24.76	34.88	46.52	71.38	86.84	102.93	0.2224	0.5338	0.4962
Panel C: All firms										
Total	104	31.90	39.49	46.90	61.76	69.49	78.07	0.2731	0.8299	0.3791

Table 4: Distribution of moments for individual firms under 7-moments models

This table reports the industry and rating distribution of our sample firms in Panels A and B, respectively. "N" represents the number of firms in each category. The 1, 3, 5, 7, 10 years credit default swap spreads are reported as basis points (*bps*). The reported values of all variables are mean values.

Rating/Industry	N	1year spread	2year spread	3year spread	5year spread	7year spread	10year spread	Implied volatility
Panel A: Rating distribution								
AAA	1	11.51	14.05	16.57	22.33	26.19	30.76	0.1743
AA	8	15.6	19.39	23.09	30.72	35.54	41.26	0.2262
A	39	17.3	21.85	26.43	35.81	41.82	48.94	0.264
BBB	47	37.94	46.28	54.7	71.91	80.68	90.29	0.2797
BB	7	83.09	104.23	121.94	154.16	166	177.62	0.3377
B	1	72.47	91.65	111.1	150.17	163.88	176.62	0.3115
CCC	1	54.54	71.96	89.48	124.22	140.64	157.33	0.2807
Panel B: Industry distribution								
Basic Materials	10	26.85	33.17	39.11	51.92	59.3	68.51	0.2807
Consumer Goods	27	32.96	40.62	48.07	62.81	70.4	78.63	0.2499
Consumer Services	17	43.87	54.62	64.65	84.1	92.76	102.56	0.2971
Energy	10	30.21	37.82	44.73	58.86	66.96	76	0.3134
Healthcare	12	20.89	26.64	32.54	44.76	51.22	58.01	0.2486
Industrials	24	29.49	35.94	42.6	56.27	63.71	71.82	0.274
Technology	3	37.66	45.74	54.12	68.55	76.46	85.5	0.2869
Telecommunications Services	1	24.76	34.88	46.52	71.38	86.84	102.93	0.2224
Panel C: All firms								
Total	104	31.76	39.32	46.7	61.52	69.23	77.79	0.2729

Table 5: Distribution of parameters with 9 moments under the Leland model

This table reports average values of parameters by fitting leverage, equity value and equity implied volatility along with the CDS spreads under the Leland model. P-values for each parameter reported in the parentheses are category level test statistics. T value reported in the table is the firm level test statistic of each parameter we estimated.

Panel A: Rating Distribution

	N	sigma	F value	J test	T value
AAA	1	0.1439	0.1632	17.2986	40.1
AA	8	0.1672	0.1590	15.6877	47.76
		(<0.0001)			
A	39	0.1675	0.1757	14.4522	63.78
		(<0.0001)			
BBB	47	0.1444	0.1750	15.3587	65.34
		(<0.0001)			
BB	7	0.1673	0.1826	13.5247	76.06
		(<0.0001)			
B	1	0.1342	0.1516	10.7666	91.88
CCC	1	0.085881394	0.1715	11.4906	51.37

Panel B: Industry Distribution

	N	sigma	F value	J test	T value
Basic Materials	10	0.1720	0.1756	13.9785	56.41
		(<0.0001)			
Consumer Goods	27	0.1505	0.1725	14.9641	63.45
		(<0.0001)			
Consumer Services	17	0.1663	0.1679	14.3626	76.6
		(<0.0001)			
Energy	10	0.1848	0.1817	12.8199	87.08
		(<0.0001)			
Healthcare	12	0.1718	0.1677	14.9742	63.97
		(<0.0001)			
Industrials	24	0.1322	0.1822	16.0571	51.45
		(<0.0001)			
Technology	3	0.1286	0.1449	17.0801	44.63
		(0.0293)			
Telecommunications Services	1	0.109646727	0.2056	12.7460	69.68

All firms

	N	sigma	F value	J test	T value

Total	104	0.1557 (<0.0001)	0.1742	14.8580	64

Table 6: Distribution of parameters with 9 moments under the CEV model

This table reports average values of parameters by fitting leverage, equity value and equity implied volatility along with the CDS spreads under the CEV model. P-values for each parameter reported in the parentheses are category level test statistics. T value reported in table is the firm level test statistic of each parameter we estimated.

Panel A: Rating distribution												
	All				Positive beta				Negative beta			
	N	sigma	beta	J test	N	sigma	beta	J test	N	sigma	beta	J test
AAA	1	0.1447	0.9973	17.0762	1	0.1447	0.9973	17.0762	0			
AA	8	0.1850	0.1355	16.6869	3	0.1622	0.8358	17.0910	5	0.1987	-0.2848	16.4445
		(<0.0001)	(0.5534)			(0.0097)	(0.0067)			(<0.0001)	(0.0701)	
A	39	0.1978	0.0107	14.3201	20	0.1690	0.5108	14.5641	19	0.2280	-0.5157	14.0632
		(<0.0001)	(0.9142)			(<0.0001)	(<0.0001)			(<0.0001)	(<0.0001)	
BBB	46	0.1621	0.1359	14.3273	25	0.1627	0.7306	14.5293	21	0.1614	-0.5722	14.0868
		(<0.0001)	(0.2434)			(<0.0001)	(<0.0001)			(<0.0001)	(<0.0001)	
BB	7	0.1937	-0.1087	13.1295	2	0.1987	0.5748	13.0681	5	0.1917	-0.3821	13.1541
		(<0.0001)	(0.6064)			(0.0051)	(0.2046)			(0.0007)	(0.0350)	
B	1	0.1133	-1.3939	11.0940	0				1	0.1133	-1.3939	11.0940
CCC	1	0.1172	0.1017	11.3541	1	0.1172	0.1017	11.3541	0			
T value												
AAA		13.28	1.7			13.28	1.7					
AA		54.72	18.2			20.32	3.94			75.36	26.76	

A		159.76	58.2			37.21	5.51			288.77	113.66	
BBB		113.12	20.84			37.08	6.06			203.64	38.44	
BB		73.8	28.32			29.65	9.25			91.46	35.95	
B		31.47	26.6							31.47	26.6	
CCC		6.81	0.5			6.81	0.5					
Panel B: Industry distribution												
	All				Positive beta				Negative beta			
	N	sigma	beta	J test	N	sigma	beta	J test	N	sigma	beta	j
Basic Materials	10	0.2088	0.3537	13.9297	6	0.1785	0.9427	14.6331	4	0.2542	-0.5294	12.8745
		(0.0006)	(0.2252)			(0.0001)	(0.0016)			(0.0899)	(0.1233)	
Consumer Goods	27	0.1603	0.0319	14.8719	13	0.1491	0.6558	14.9825	14	0.1707	-0.5474	14.7692
		(<0.0001)	(0.8159)			(<0.0001)	(<0.0001)			(<0.0001)	(0.0001)	
Consumer Services	17	0.1836	0.0805	13.9268	10	0.1681	0.5161	13.9285	7	0.2058	-0.5417	13.9243
		(<0.0001)	(0.6380)			(<0.0001)	(0.0067)			(<0.0001)	(0.0157)	
Energy	10	0.2194	-0.1670	12.4489	4	0.2062	0.2551	12.9927	6	0.2282	-0.4484	12.0864
		(<0.0001)	(0.2670)			(0.0078)	(0.0675)			(0.0012)	(0.0182)	
Healthcare	12	0.1891	0.2169	15.4313	5	0.1689	0.9208	15.0928	7	0.2036	-0.2859	15.6731
		(<0.0001)	(0.2864)			(0.0002)	(<0.0001)			(<0.0001)	(0.0631)	
Industrials	23	0.1594	-0.1138	14.5958	11	0.1513	0.5054	15.2057	12	0.1668	-0.6813	14.0368
		(<0.0001)	(0.4514)			(<0.0001)	(0.0008)			(<0.0001)	(0.0001)	
Technology	3	0.1875	0.5006	15.5814	2	0.2090	0.8014	15.5213	1	0.1444	-0.101	15.7017
		(0.0152)	(0.2398)			(0.0483)	(0.0444)					
Telecommunications Services	1	0.1551	1.1093	12.7566	1	0.1551	1.1093	12.7566				

T value

Basic Materials		64.13	20.22			40.27	9.18			99.92	36.77	
Consumer Goods		130.74	19.22			29.73	4.7			224.54	32.71	
Consumer Services		278.67	80.21			36.44	5.04			624.7	187.6	
Energy		82.76	24.97			43.87	4.86			108.69	38.38	
Healthcare		60.65	42.6			25.45	5.89			85.8	68.82	
Industrials		75.86	31.42			32.49	4.9			115.62	55.74	
Technology		69.98	9.05			68.87	9.68			72.2	7.77	
Telecommunications Services		21.25	5.29			21.25	5.29					
Panel C: All firms												
	All				Positive beta				Negative beta			
	N	sigma	beta	J test	N	sigma	beta	J test	N	sigma	beta	J test
Total	103	0.1785	0.0650	14.3928	52	0.1653	0.6392	14.6222	51	0.1919	-0.5204	14.1590
		(<0.0001)	(0.3471)			(<0.0001)	(<0.0001)			(<0.0001)	(<0.0001)	
T value												
		120.78	34.96			34.84	5.66			208.4	64.84	

Table 7: Distribution of parameters with 7 moments under the Leland model

This table reports average values of parameters by fitting equity implied volatility along with the CDS spreads under the Leland model. P-values for each parameter reported in the parentheses are category level test statistics. T value reported in table is the firm level test statistic of each parameter we estimated.

Panel A: Rating Distribution

	N	sigma	F value	J test	T value
AAA	1	0.1407	0.1482	15.7082	38.98
AA	8	0.1732 (<0.0001)	0.1473	14.5199	46.9
A	39	0.1743 (<0.0001)	0.1662	13.6492	44.55
BBB	47	0.1494 (<0.0001)	0.1660	14.6625	34.55
BB	7	0.1821 (<0.0001)	0.1668	12.3731	48.1
B	1	0.1254	0.1519	10.7862	42.1
CCC	1	0.0816	0.1946	13.0372	39.83

Panel B: Industry Distribution

	N	sigma	F value	J test	T value
Basic Materials	10	0.1729 (<0.0001)	0.1703	13.5040	41.14
Consumer Goods	27	0.1563 (<0.0001)	0.1671	14.5313	38.73
Consumer Services	17	0.1782 (<0.0001)	0.1602	13.6668	42.68
Energy	10	0.1931 (<0.0001)	0.1680	11.8808	41.29
Healthcare	12	0.1773 (<0.0001)	0.1570	13.9359	48.95
Industrials	24	0.1359 (<0.0001)	0.1675	14.8851	35.01
Technology	3	0.1404 (0.0228)	0.1353	16.1689	38.83
Telecommunications Services	1	0.1082	0.1976	12.2482	54.46

All firms

	sigma	F value	J test	T value

Total	104	0.1618 (<0.0001)	0.1646	14.0746	40.33

Table 8: Distribution of parameter with 7 moments under the CEV model

This table reports average values of parameters by fitting equity implied volatility along with the CDS spreads under the CEV model. P-values for each parameter reported in the parentheses are category level test statistics. T value reported in table is the firm level test statistic of each parameter we estimated.

Panel A: Rating distribution												
	All				Positive beta				Negative beta			
	N	sigma	beta	J test	N	sigma	beta	J test	N	sigma	beta	J test
AAA	1	0.1639	-0.9688	16.484	0				1	0.1639	-0.9688	16.484
AA	8	0.1899 (<0.0001)	-0.4820 (0.0271)	15.8734	2	0.2016 (0.0303)	0.1	14.6972	6	0.1860 (<0.0001)	-0.6761 (0.0084)	16.2655
A	38	0.2198 (<0.0001)	-0.5197 (<0.0001)	13.2864	11	0.1858 (<0.0001)	0.3989 (0.0015)	13.2885	27	0.2336 (<0.0001)	-0.8940 (<0.0001)	13.2856
BBB	47	0.1858 (<0.0001)	-0.5090 (0.0001)	13.6437	13	0.1555 (<0.0001)	0.7139 (0.0005)	14.2449	34	0.1973 (<0.0001)	-0.9766 (<0.0001)	13.4138
BB	7	0.2175 (<0.0001)	-0.7865 (0.1521)	11.0514	2	0.1911 (0.1131)	0.5574 (0.4270)	12.0651	5	0.2281 (0.0005)	-1.3240 (0.0448)	10.6460
B	1	0.1213	-1.3682	12.3008	0				1	0.1213	-1.3682	12.3008
CCC	1	0.1261	0.1005	12.5876	1	0.1261	0.1005	12.5876	0			
T value												
AAA		28.75	169.96							28.75	169.96	
AA		37.21	88.85			28.71	14.39			40.04	113.66	
A		43.59	184.36			25.95	64.13			50.78	233.34	
BBB		33.6	167.24			20.27	77.44			38.69	201.57	

BB		30.92	106.04			24.31	22.83			34.22	147.65	
B		19.89	224.3							19.89	224.30	
CCC		6.21	4.95			6.21	4.95					
Panel B: Industry distribution												
	All				Positive beta				Negative beta			
	N	sigma	beta	J test	N	sigma	beta	J test	N	sigma	beta	J test
Basic Materials	10	0.2291	-0.8618	13.2281	1	0.1751	1.0414	15.1523	9	0.2350	-1.0733	13.0143
		(0.0006)	(0.0032)							(0.0013)	(<0.0001)	
Consumer Goods	27	0.1796	-0.5385	13.9951	8	0.1524	0.7725	14.9371	19	0.1911	-1.0906	13.5985
		(<0.0001)	(0.0112)			(<0.0001)	(0.0091)			(<0.0001)	(<0.0001)	
Consumer Services	17	0.2016	-0.2254	13.1237	9	0.1780	0.3621	13.1865	8	0.2282	-0.8863	13.0530
		(<0.0001)	(0.1974)			(<0.0001)	(0.0102)			(<0.0001)	(<0.0001)	
Energy	9	0.2683	-0.6084	11.4854	1	0.2403	0.1605	10.6878	8	0.2718	-0.7045	11.5851
		(<0.0001)	(0.0006)							(<0.0001)	(<0.0001)	
Healthcare	12	0.1965	-0.2919	15.0803	4	0.1917	0.5998	14.6710	8	0.1989	-0.7378	15.2849
		(<0.0001)	(0.2241)			(0.0007)	(0.0762)			(<0.0001)	(0.0023)	
Industrials	24	0.1925	-0.7108	13.5601	4	0.1590	0.4084	13.1652	20	0.1992	-0.9347	13.6391
		(<0.0001)	(<0.0001)			(0.0021)	(0.1377)			(<0.0001)	(<0.0001)	
Technology	3	0.1492	-0.7947	12.2032	1	0.2099	0.2702	9.886	2	0.1189	-1.3271	13.3618
		(0.0451)	(0.2900)							(0.1142)	(0.1348)	
Telecommunications Services	1	0.1282	0.1393	12.2224	1	0.1282	0.1393	12.2224				
T value												
Basic Materials		28.41	172.44			13.37	79.50			30.08	182.77	
Consumer Goods		49.73	245.85			19.41	88.86			63.20	315.62	
Consumer Services		29.67	97.42			21.99	46.80			38.30	154.37	
Energy		31.44	75.18			26.41	17.64			32.07	82.38	

Healthcare		34.19	108.18			20.38	49.58			41.10	137.49	
Industrials		35.26	156.92			26.16	65.21			37.07	175.26	
Technology		46.98	361.80			69.97	90.07			35.49	497.66	
Telecommunications Services		12.09	13.14			12.09	13.14					
Panel C: All firms												
	All				Positive beta				Negative beta			
	N	sigma	beta	J test	N	sigma	beta	J test	N	sigma	beta	J test
Total	103	0.1994 (<0.0001)	-0.5366 (<0.0001)	13.5132	29	0.1716 (<0.0001)	0.5201 (<0.0001)	13.7058	74	0.2103 (<0.0001)	-0.9508 (<0.0001)	13.4377
T value												
		37	162.86			22.80	61.78			42.64	203.02	

Table 9: Distribution of Sigma across different firms for the Leland model

This table presents the distribution of Sigma across individual firms for the Leland model, and for both 7-moments and 9-moments. For 7-moments, the firm "Dow Chem Co" has the largest Sigma 0.2974, the firm "Raytheon Co" has the smallest Sigma 0.05, and the average value of Sigma is 0.1618. For 9-moments, the firm "Dow Chem Co" has the largest Sigma 0.2913, the firm "Raytheon Co" has the smallest Sigma 0.05, and the average value of Sigma is 0.1557.

Firm	Sigma_7mom	Sigma_9mom	Firm	Sigma_7mom	Sigma_9mom
Abbott Labs	0.1842	0.1567	Norfolk Sthn Corp	0.1282	0.1211
Air Prods & Chems Inc	0.1551	0.1564	Northrop Grumman Corp	0.1099	0.1061
Honeywell Intl Inc	0.1573	0.1598	OCCIDENTAL PETROLEUM CORP	0.2008	0.1929
Alcoa Inc.	0.1947	0.1893	Omnicare Inc	0.2000	0.1788
Wyeth	0.1869	0.1833	PPG Inds Inc	0.1524	0.1428
Anheuser Busch Cos Inc	0.1382	0.1394	J C Penney Co Inc	0.1935	0.1474
APACHE CORP	0.2179	0.2195	Pepsico Inc	0.1388	0.1327
Archer Daniels Midland Co	0.1754	0.1519	Pfizer Inc	0.1637	0.1655
Arrow Electrs Inc	0.1393	0.1134	Altria Gp Inc	0.2867	0.2443
Avon Prods Inc	0.2170	0.2175	ConocoPhillips	0.1358	0.1285
Baker Hughes Inc	0.2756	0.2749	Pitney Bowes Inc	0.0976	0.0847
Baxter Intl Inc	0.1527	0.1210	Procter & Gamble Co	0.1509	0.1513
Black & Decker Corp	0.1435	0.1428	Raytheon Co	0.0500	0.0500
Boeing Co	0.1895	0.1834	Rohm & Haas Co	0.1389	0.1633
Bristol Myers Squibb Co	0.1865	0.1818	Ryder Sys Inc	0.1352	0.1302
CSX Corp	0.0804	0.0886	Safeway Inc	0.1366	0.1316
Campbell Soup Co	0.1489	0.1330	Schering Plough Corp	0.2203	0.2200

Caterpillar Inc	0.1259	0.1153	Sealed Air Corp US	0.1254	0.1342
CenturyTel Inc	0.1082	0.1096	Sherwin Williams Co	0.1867	0.1875
Clorox Co	0.1513	0.1520	Smithfield Foods Inc	0.1253	0.1240
Colgate Palmolive Co	0.1422	0.1457	Southwest Airls Co	0.1776	0.1699
ConAgra Foods Inc	0.1147	0.1160	Sunoco Inc	0.1727	0.1505
Molson Coors Brewing Co	0.1619	0.1522	SUPERVALU INC	0.0816	0.0859
Danaher Corp	0.1921	0.1914	Sysco Corp	0.1684	0.1609
Target Corp	0.1979	0.1917	Textron Inc	0.0685	0.0507
Dover Corp	0.1551	0.1460	Un Pac Corp	0.1131	0.1256
Dow Chem Co	0.2974	0.2913	Utd Parcel Svc Inc	0.1389	0.1437
Omnicom Gp Inc	0.1321	0.1257	UST Inc.	0.1802	0.1784
FedEx Corp	0.1938	0.1882	Utd Tech Corp	0.1387	0.1407
Gen Dynamics Corp	0.1429	0.1354	Unvl Health Svcs Inc	0.1621	0.1603
Gen Mls Inc	0.1019	0.1063	V F Corp	0.1954	0.1936
Goodrich Corp	0.1394	0.1320	Wal Mart Stores Inc	0.2033	0.1836
Halliburton Co	0.2444	0.2147	Whirlpool Corp	0.1216	0.1164
H J HEINZ CO	0.1174	0.1181	Anadarko Pete Corp	0.1599	0.1569
Home Depot Inc	0.2515	0.2170	Coca Cola Entpers Inc	0.0667	0.0665
Intl Business Machs Corp	0.1667	0.1592	Waste Mgmt Inc	0.1241	0.1267
Intl Paper Co	0.1049	0.1064	Pride Intl Inc	0.2466	0.2306
Johnson & Johnson	0.1407	0.1439	Autozone Inc	0.1821	0.1718
Kellogg Co	0.1279	0.1220	Mohawk Inds Inc	0.1950	0.1913
Kimberly Clark Corp	0.1260	0.1088	Praxair Inc	0.1704	0.1734
The Kroger Co.	0.1177	0.1148	BorgWarner Inc	0.1819	0.1728
Eli Lilly & Co	0.1823	0.1811	ONEOK Partners LP	0.0924	0.0870
Ltd Brands Inc	0.2341	0.2246	Marriott Intl Inc	0.2091	0.2037
Lockheed Martin Corp	0.1352	0.1261	Costco Whsl Corp	0.1740	0.1645

Lowes Cos Inc	0.2216	0.2202	Eastman Chem Co	0.1073	0.1064
Masco Corp	0.1868	0.1853	Cytec Inds Inc	0.1714	0.1643
Medtronic Inc	0.1896	0.1888	AmerisourceBergen Corp	0.0828	0.0884
Merck & Co Inc	0.1954	0.1959	Diamond Offshore Drilling Inc	0.1850	0.1929
3M Co	0.1794	0.1797	Quest Diagnostics Inc	0.1633	0.1638
Motorola Inc	0.1568	0.1419	Rep Svcs Inc	0.1214	0.1140
Newell Rubbermaid Inc	0.1698	0.1566	Reynolds Amern Inc	0.1672	0.1578
Nordstrom Inc	0.2440	0.2170	Monsanto Co	0.2365	0.2265

Table 10: Distribution of Sigma across A and BBB rated firms for the Leland model

This table presents the Sigma distribution for A and BBB rated firms. The average Sigma value for A rated firms with 7- and 9-moments are 0.1743 and 0.1675 respectively; The average Sigma value for BBB rated firms with 7- and 9-moments are 0.1494 and 0.1444 respectively; The total average Sigma value for these firms with 7- and 9-moments are 0.1607 and 0.1549 respectively. For A rated firms, “Baker Hughes Inc” has the largest Sigma value of 0.2756 and 0.2749 for 7- and 9-moments respectively; “Raytheon Co” has the smallest Sigma value of 0.05 and 0.05 for 7- and 9-moments respectively. For BBB rating firms, “Dow Chem Co” has the largest Sigma value of 0.2974 and 0.2913 for 7- and 9-moments respectively; “Textron Inc” has the smallest Sigma value of 0.0685 and 0.0507 for 7- and 9-momentsrespectively.

Rating A				Rating BBB			
Firm	**Rating**	**Sigma_7mom**	**sigma_9mom**	**Firm**	**Rating**	**Sigma_7mom**	**sigma_9mom**
Baker Hughes Inc	A	0.2756	0.2749	Dow Chem Co	BBB	0.2974	0.2913
Home Depot Inc	A	0.2515	0.2170	Alcoa Inc.	BBB	0.1947	0.1893
Halliburton Co	A	0.2444	0.2147	FedEx Corp	BBB	0.1938	0.1882
Nordstrom Inc	A	0.2440	0.2170	Mohawk Inds Inc	BBB	0.1950	0.1913
Monsanto Co	A	0.2365	0.2265	Altria Gp Inc	BBB	0.2867	0.2443
Lowes Cos Inc	A	0.2216	0.2202	Pride Intl Inc	BBB	0.2466	0.2306
Schering Plough Corp	A	0.2203	0.2200	Avon Prods Inc	BBB	0.2170	0.2175
APACHE CORP	A	0.2179	0.2195	Marriott Intl Inc	BBB	0.2091	0.2037
OCCIDENTAL PETROLEUM CORP	A	0.2008	0.1929	BorgWarner Inc	BBB	0.1819	0.1728
Target Corp	A	0.1979	0.1917	Southwest Airls Co	BBB	0.1776	0.1699
V F Corp	A	0.1954	0.1936	UST Inc.	BBB	0.1802	0.1784

Danaher Corp	A	0.1921	0.1914	Cytec Inds Inc	BBB	0.1714	0.1643
Medtronic Inc	A	0.1896	0.1888	Quest Diagnostics Inc	BBB	0.1633	0.1638
Boeing Co	A	0.1895	0.1834	Reynolds Amern Inc	BBB	0.1672	0.1578
Wyeth	A	0.1869	0.1833	Anadarko Pete Corp	BBB	0.1599	0.1569
Sherwin Williams Co	A	0.1867	0.1875	Rohm & Haas Co	BBB	0.1389	0.1633
Bristol Myers Squibb Co	A	0.1865	0.1818	Newell Rubbermaid Inc	BBB	0.1698	0.1566
Diamond Offshore Drilling Inc	A	0.1850	0.1929	Autozone Inc	BBB	0.1821	0.1718
Eli Lilly & Co	A	0.1823	0.1811	Molson Coors Brewing Co	BBB	0.1619	0.1522
Archer Daniels Midland Co	A	0.1754	0.1519	Safeway Inc	BBB	0.1366	0.1316
Costco Whsl Corp	A	0.1740	0.1645	PPG Inds Inc	BBB	0.1524	0.1428
Praxair Inc	A	0.1704	0.1734	Clorox Co	BBB	0.1513	0.1520
Sysco Corp	A	0.1684	0.1609	Black & Decker Corp	BBB	0.1435	0.1428
Honeywell Intl Inc	A	0.1573	0.1598	Motorola Inc	BBB	0.1568	0.1419
Dover Corp	A	0.1551	0.1460	Ryder Sys Inc	BBB	0.1352	0.1302
Air Prods & Chems Inc	A	0.1551	0.1564	Goodrich Corp	BBB	0.1394	0.1320
Baxter Intl Inc	A	0.1527	0.1210	The Kroger Co.	BBB	0.1177	0.1148
Campbell Soup Co	A	0.1489	0.1330	Norfolk Sthn Corp	BBB	0.1282	0.1211
Gen Dynamics Corp	A	0.1429	0.1354	Kellogg Co	BBB	0.1279	0.1220
Utd Parcel Svc Inc	A	0.1389	0.1437	Waste Mgmt Inc	BBB	0.1241	0.1267
Pepsico Inc	A	0.1388	0.1327	H J HEINZ CO	BBB	0.1174	0.1181
Utd Tech Corp	A	0.1387	0.1407	Omnicom Gp Inc	BBB	0.1321	0.1257
Anheuser Busch Cos Inc	A	0.1382	0.1394	Un Pac Corp	BBB	0.1131	0.1256
ConocoPhillips	A	0.1358	0.1285	Whirlpool Corp	BBB	0.1216	0.1164
Lockheed Martin Corp	A	0.1352	0.1261	ConAgra Foods Inc	BBB	0.1147	0.1160
Kimberly Clark Corp	A	0.1260	0.1088	Arrow Electrs Inc	BBB	0.1393	0.1134
Caterpillar Inc	A	0.1259	0.1153	Rep Svcs Inc	BBB	0.1214	0.1140

Coca Cola Entpers Inc	A	0.0667	0.0665	CenturyTel Inc	BBB	0.1082	0.1096
Raytheon Co	A	0.0500	0.0500	Eastman Chem Co	BBB	0.1073	0.1064
				Intl Paper Co	BBB	0.1049	0.1064
				Gen Mls Inc	BBB	0.1019	0.1063
				Northrop Grumman Corp	BBB	0.1099	0.1061
				CSX Corp	BBB	0.0804	0.0886
				AmerisourceBergen Corp	BBB	0.0828	0.0884
				ONEOK Partners LP	BBB	0.0924	0.0870
				Pitney Bowes Inc	BBB	0.0976	0.0847
				Textron Inc	BBB	0.0685	0.0507
	Rating A				**Rating BBB**		
		7 moment	**9 momnet**			**7 moment**	**9 moment**
Average		0.1743	0.1675			0.1494	0.1444
		7 moment	**9 momnet**				
Total Average		0.1607	0.1549				

Table 11: Beta distribution for CEV model with 9-moment conditions

This table presents the beta distribution for CEV model with 9-moment conditions. There are 51 firms with negative beta and 52 firms with positive beta. Panel A shows the firms with negative betas. Firm “Gen Mls Inc” has the smallest beta. The average beta, average J statistic, and average T value are -0.5204, 14.159 and 64.8393 respectively for the negative group; panel B shows the firms with positive betas. Firm “AmerisourceBergen Corp” has the largest beta. The average beta, average J statistic, and average T value are 0.6392, 14.6222 and 5.656 respectively for the positive group.

Firm	Beta	Standard error	J Test	T
Negative beta				
Gen Mls Inc	-1.4099	0.0685	16.0398	20.5825
Sealed Air Corp US	-1.3939	0.0524	11.0940	26.6011
The Kroger Co.	-1.0664	0.0807	12.7630	13.2144
Lowes Cos Inc	-1.0215	0.0200	14.0085	51.0750
Intl Paper Co	-1.0119	0.0505	14.6964	20.0376
Medtronic Inc	-1.0091	0.0319	16.1936	31.6332
Honeywell Intl Inc	-0.9650	0.0443	17.3969	21.7833
Textron Inc	-0.9457	0.0275	12.4459	34.3891
Ryder Sys Inc	-0.9225	0.0273	12.0790	33.7912
OCCIDENTAL PETROLEUM CORP	-0.9067	0.0176	12.3581	51.5170
Monsanto Co	-0.9053	0.0422	11.4217	21.4526
Pepsico Inc	-0.8975	0.0424	15.2876	21.1675
Caterpillar Inc	-0.8855	0.0063	14.8702	140.5556

Mohawk Inds Inc	-0.8650	0.0362	13.9250	23.8950
Norfolk Sthn Corp	-0.8525	0.0526	14.4680	16.2072
Omnicare Inc	-0.8358	0.0457	12.2683	18.2888
H J HEINZ CO	-0.7942	0.0719	18.0136	11.0459
Un Pac Corp	-0.7770	0.0332	12.4555	23.4036
Waste Mgmt Inc	-0.6778	0.0523	11.4072	12.9598
Archer Daniels Midland Co	-0.6696	0.0242	11.1345	27.6694
Anheuser Busch Cos Inc	-0.6653	0.0334	12.8858	19.9192
Procter & Gamble Co	-0.6456	0.0287	18.4776	22.4948
Pride Intl Inc	-0.6218	0.0156	11.1393	39.8590
Halliburton Co	-0.6108	0.0409	11.5637	14.9340
Molson Coors Brewing Co	-0.5213	0.0485	12.8712	10.7485
Wal Mart Stores Inc	-0.4772	0.0669	18.4947	7.1330
Masco Corp	-0.3785	0.0303	12.7038	12.4917
UST Inc.	-0.3609	0.0338	13.3902	10.6775
Schering Plough Corp	-0.3270	0.0187	13.3983	17.4866
Sunoco Inc	-0.3220	0.0032	12.2454	100.6250
Southwest Airls Co	-0.3060	0.0115	16.4563	26.6087
Unvl Health Svcs Inc	-0.2632	0.0063	15.7583	41.7778
Goodrich Corp	-0.2080	0.0010	13.5138	208.0000
Target Corp	-0.1683	0.0116	13.2191	14.5086
Utd Parcel Svc Inc	-0.1411	0.0023	14.9576	61.3478
ConAgra Foods Inc	-0.1361	0.0511	17.0622	2.6634
ONEOK Partners LP	-0.1292	0.0101	13.6648	12.7921
Sysco Corp	-0.1197	0.0001	13.7570	1157.6402
Smithfield Foods Inc	-0.1108	0.0169	12.7948	6.5562
Altria Gp Inc	-0.1088	0.0362	17.2010	3.0055

Home Depot Inc	-0.1027	0.0020	12.9598	51.3500
Eli Lilly & Co	-0.1022	0.0063	17.3523	16.2222
3M Co	-0.1011	0.0016	17.2972	63.1875
Pitney Bowes Inc	-0.1010	0.0130	15.7017	7.7692
Praxair Inc	-0.1002	0.0045	12.3330	22.2667
Baxter Intl Inc	-0.1001	0.0003	19.0565	333.6667
Air Prods & Chems Inc	-0.1000	0.0012	13.0470	83.3333
Black & Decker Corp	-0.1000	0.0004	14.9817	264.9709
Merck & Co Inc	-0.1000	0.0046	13.6090	21.7391
Pfizer Inc	-0.1000	0.0052	14.3440	19.2308
Anadarko Pete Corp	-0.1000	0.0095	11.5471	10.5263
Average	-0.5204	0.0263	14.1590	64.8393
Positive beta				
APACHE CORP	0.1000	0.0694	13.6553	1.4409
Baker Hughes Inc	0.1000	0.0322	13.7164	3.1056
CSX Corp	0.1000	0.1070	11.7133	0.9346
Danaher Corp	0.1000	0.0971	16.0142	1.0299
Kellogg Co	0.1000	0.0830	17.1015	1.2048
Autozone Inc	0.1000	0.2091	16.8616	0.4782
SUPERVALU INC	0.1017	0.2014	11.3541	0.5050
Safeway Inc	0.1077	0.0613	13.0120	1.7569
Northrop Grumman Corp	0.1587	0.1455	16.1149	1.0907
Marriott Intl Inc	0.2156	0.0776	13.1209	2.7784
Raytheon Co	0.2271	0.1511	17.1213	1.5030
Coca Cola Entpers Inc	0.3032	0.3190	13.0901	0.9505
FedEx Corp	0.3090	0.0632	14.8316	4.8892

V F Corp	0.3279	0.0788	14.3328	4.1612
BorgWarner Inc	0.3583	0.1038	13.8510	3.4518
Sherwin Williams Co	0.3625	0.0910	15.8362	3.9835
Cytec Inds Inc	0.3684	0.1118	15.4304	3.2952
Diamond Offshore Drilling Inc	0.3716	0.0328	12.8220	11.3293
J C Penney Co Inc	0.3835	0.1103	14.8752	3.4769
Nordstrom Inc	0.4221	0.0532	13.5369	7.9342
ConocoPhillips	0.4487	0.1262	11.7771	3.5555
Costco Whsl Corp	0.5342	0.1020	14.1839	5.2373
Boeing Co	0.5382	0.1491	14.5554	3.6097
Dover Corp	0.5983	0.0622	14.1908	9.6190
Utd Tech Corp	0.6000	0.0626	12.9556	9.5847
Campbell Soup Co	0.6002	0.1118	16.0267	5.3685
Avon Prods Inc	0.6760	0.1113	16.4972	6.0737
Dow Chem Co	0.6994	0.0767	13.8217	9.1186
Intl Business Machs Corp	0.7454	0.1542	18.5198	4.8340
Ltd Brands Inc	0.7662	0.0510	11.2610	15.0235
Abbott Labs	0.7915	0.1393	16.3636	5.6820
Motorola Inc	0.8574	0.0590	12.5227	14.5322
Bristol Myers Squibb Co	0.8709	0.0883	17.4791	9.8630
Lockheed Martin Corp	0.8879	0.1731	18.0761	5.1294
PPG Inds Inc	0.9065	0.1036	17.0506	8.7500
Wyeth	0.9191	0.1524	12.4004	6.0308
Gen Dynamics Corp	0.9402	0.0846	14.5436	11.1135
Whirlpool Corp	0.9476	0.2302	14.6319	4.1164
Newell Rubbermaid Inc	0.9496	0.0569	15.9277	16.6889
Kimberly Clark Corp	0.9637	0.1709	14.9676	5.6390

Reynolds Amern Inc	0.9655	0.2891	14.3849	3.3397
Colgate Palmolive Co	0.9706	0.7450	16.3896	1.3028
Johnson & Johnson	0.9973	0.5879	17.0762	1.6964
Omnicom Gp Inc	0.9975	0.1457	16.3600	6.8463
Clorox Co	1.0005	0.2056	11.7352	4.8662
Quest Diagnostics Inc	1.0254	0.1656	12.1446	6.1920
Alcoa Inc.	1.0903	0.0546	14.4232	19.9689
Arrow Electrs Inc	1.0995	0.2044	17.1461	5.3792
CenturyTel Inc	1.1093	0.2095	12.7566	5.2950
Eastman Chem Co	1.1499	0.1531	11.8492	7.5108
Rohm & Haas Co	1.4415	0.2232	15.2233	6.4583
AmerisourceBergen Corp	1.5320	0.2398	14.7193	6.3887
Average	0.6392	0.1478	14.6222	5.6560

Table 12: Beta distribution for the CEV model with 7-moment conditions

This table presents the beta distribution for the CEV model with 7-moment conditions. There are 74 firms with negative beta and 29 firms with positive beta. Panel A shows the firms with negative betas. Firm "Smithfield Foods Inc" has the smallest beta. The average beta, average J statistic, and average T value are-0.9508, 13.4377 and 200.7618 respectively for the negative group; panel B shows the firms with positive betas. Firm "Gen Mls Inc" has the largest beta. The average beta, average J statistic, and average T value are 0.5201, 13.7058 and 61.777 respectively for the positive group.

Firm	Beta	Standard error	J Test	T
Negative beta				
Smithfield Foods Inc	-3.0525	0.0851	10.7882	35.8696
Pitney Bowes Inc	-1.6123	0.0596	9.8951	848.5789
Masco Corp	-1.4593	0.0645	9.6145	291.8600
Quest Diagnostics Inc	-1.4319	0.0459	12.1357	318.2000
Intl Paper Co	-1.4047	0.0665	13.8871	401.3429
Reynolds Amern Inc	-1.3858	0.0391	12.5708	230.9667
Sealed Air Corp US	-1.3682	0.0693	12.3008	224.2951
Raytheon Co	-1.2901	0.0788	14.9110	222.4310
Eastman Chem Co	-1.2259	0.0775	11.2071	215.0702
Pepsico Inc	-1.1710	0.0594	14.4490	249.1489
Dow Chem Co	-1.1502	0.0702	13.1101	426.0000
Un Pac Corp	-1.1016	0.1119	11.8393	121.0549
Autozone Inc	-1.0999	0.0469	17.0106	224.4694

Lowes Cos Inc	-1.0823	0.0300	13.9755	300.6389
Kellogg Co	-1.0744	0.0956	17.2310	358.1333
Black & Decker Corp	-1.0572	0.0508	13.7356	160.1818
Molson Coors Brewing Co	-1.0524	0.0989	11.2845	113.1613
Lockheed Martin Corp	-1.0495	0.0775	14.1631	276.1842
V F Corp	-1.0479	0.0429	12.1260	2095.8000
Cytec Inds Inc	-1.0479	0.0465	14.7499	100.7596
Anheuser Busch Cos Inc	-1.0453	0.0576	12.6176	201.0192
Intl Business Machs Corp	-1.0419	0.0722	16.8284	146.7465
Medtronic Inc	-1.0410	0.0328	14.8923	185.8929
Coca Cola Entpers Inc	-1.0390	0.0733	15.5321	611.1765
Colgate Palmolive Co	-1.0292	0.0496	16.2404	177.4483
Wyeth	-1.0271	0.0690	14.5739	180.1930
Gen Dynamics Corp	-1.0094	0.0524	13.6122	142.1690
Alcoa Inc.	-1.0073	0.0681	14.7320	111.9222
Northrop Grumman Corp	-1.0035	0.0535	15.0197	346.0345
Utd Tech Corp	-0.9985	0.0378	12.1775	129.6753
Air Prods & Chems Inc	-0.9932	0.0636	10.8773	91.9630
Ryder Sys Inc	-0.9912	0.0534	11.8448	120.8780
Sherwin Williams Co	-0.9816	0.0625	14.0872	103.3263
Monsanto Co	-0.9742	0.0491	10.9551	35.4255
Johnson & Johnson	-0.9688	0.0606	16.4840	169.9649
Pride Intl Inc	-0.9673	0.0500	11.1092	89.5648
The Kroger Co.	-0.9651	0.0718	12.5403	229.7857
Textron Inc	-0.9580	0.0726	11.4960	177.4074
FedEx Corp	-0.9557	0.0119	16.4899	66.8322
Marriott Intl Inc	-0.9522	0.0535	12.4622	78.6942

PPG Inds Inc	-0.9435	0.0334	16.6418	159.9153
H J HEINZ CO	-0.9138	0.0513	19.7234	179.1765
Praxair Inc	-0.9125	0.0455	10.9687	102.5281
Procter & Gamble Co	-0.9114	0.0530	15.4600	136.0299
Archer Daniels Midland Co	-0.9067	0.0803	11.0206	159.0702
Altria Gp Inc	-0.9064	0.0816	12.4950	95.4105
Rep Svcs Inc	-0.9052	0.0372	14.2439	100.5778
Goodrich Corp	-0.8988	0.0521	13.0832	187.2500
Danaher Corp	-0.8927	0.0420	15.2366	111.5875
OCCIDENTAL PETROLEUM CORP	-0.8911	0.0433	11.7143	64.5725
Omnicare Inc	-0.8541	0.0406	11.8905	94.9000
Southwest Airls Co	-0.8198	0.0424	15.3991	174.4255
ConocoPhillips	-0.8124	0.0270	11.7337	116.0571
Honeywell Intl Inc	-0.8088	0.0653	14.5204	149.7778
Caterpillar Inc	-0.7819	0.0426	12.8130	190.7073
Safeway Inc	-0.7657	0.0728	12.2633	104.8904
ONEOK Partners LP	-0.7655	0.0856	13.2526	74.3204
Norfolk Sthn Corp	-0.7629	0.0232	12.2674	231.1818
Boeing Co	-0.7290	0.0426	12.8739	220.9091
Newell Rubbermaid Inc	-0.7243	0.0711	13.2427	109.7424
J C Penney Co Inc	-0.7068	0.0597	11.4594	102.4348
CSX Corp	-0.7009	0.0120	11.1823	194.6944
Anadarko Pete Corp	-0.6910	0.0333	11.7730	93.3784
3M Co	-0.6675	0.0572	17.3084	117.1053
Target Corp	-0.6643	0.0273	12.8219	99.1493
BorgWarner Inc	-0.6277	0.0369	12.5538	313.8500

Sunoco Inc	-0.5475	0.1004	9.4773	101.3889
Baxter Intl Inc	-0.5363	0.0588	19.6418	75.5352
Schering Plough Corp	-0.4909	0.0341	12.7961	65.4533
Baker Hughes Inc	-0.4875	0.0277	13.2236	79.9180
APACHE CORP	-0.4739	0.0250	10.3974	39.8235
UST Inc.	-0.3349	0.0240	13.5983	95.6857
Pfizer Inc	-0.2846	0.0400	18.6323	51.7455
Merck & Co Inc	-0.1217	0.0104	13.1233	52.9130
Average	-0.9508	0.0542	13.4377	200.7618
		Positive beta		
Abbott Labs	0.1000	0.1090	13.8598	17.5439
Wal Mart Stores Inc	0.1000	0.0799	15.5346	11.2360
SUPERVALU INC	0.1005	0.2103	12.5876	4.9507
Ltd Brands Inc	0.1152	0.0810	10.2125	23.0400
Waste Mgmt Inc	0.1154	0.2223	11.1237	10.3964
Home Depot Inc	0.1294	0.0660	10.7552	8.6846
Dover Corp	0.1303	0.1004	12.4872	21.7167
CenturyTel Inc	0.1393	0.3129	12.2224	13.1415
Mohawk Inds Inc	0.1549	0.2042	13.3039	13.1271
Halliburton Co	0.1605	0.0554	10.6878	17.6374
Sysco Corp	0.2003	0.0877	12.3173	22.2556
Campbell Soup Co	0.2342	0.1086	15.1319	38.3934
Motorola Inc	0.2702	0.0874	9.8860	90.0667
Eli Lilly & Co	0.3378	0.0881	14.1718	22.6711
Nordstrom Inc	0.3855	0.0656	13.2957	66.4655
Avon Prods Inc	0.3963	0.1130	15.7842	68.3276

Utd Parcel Svc Inc	0.4040	0.1249	12.0812	126.2500
Costco Whsl Corp	0.4623	0.1418	13.7251	57.0741
ConAgra Foods Inc	0.5029	0.1863	16.1409	34.9236
Omnicom Gp Inc	0.7960	0.3310	15.6059	104.7368
Clorox Co	0.8742	0.3979	11.4175	136.5938
Bristol Myers Squibb Co	0.9619	0.1162	16.7348	135.4789
AmerisourceBergen Corp	0.9697	0.4939	14.6450	122.7468
Kimberly Clark Corp	0.9816	0.2035	14.7852	188.7692
Whirlpool Corp	0.9819	0.3237	14.4610	78.5520
Arrow Electrs Inc	0.9838	0.1927	16.9686	102.4792
Unvl Health Svcs Inc	0.9996	1.1247	13.9177	22.6154
Rohm & Haas Co	1.0414	2.1474	15.1523	79.4962
Gen Mls Inc	2.0542	1.1858	18.4723	152.1630
Average	0.5201	0.3090	13.7058	61.7770

Figure 1: Time series of the 5-year CDS spreads over the period from 2001 to 2011.

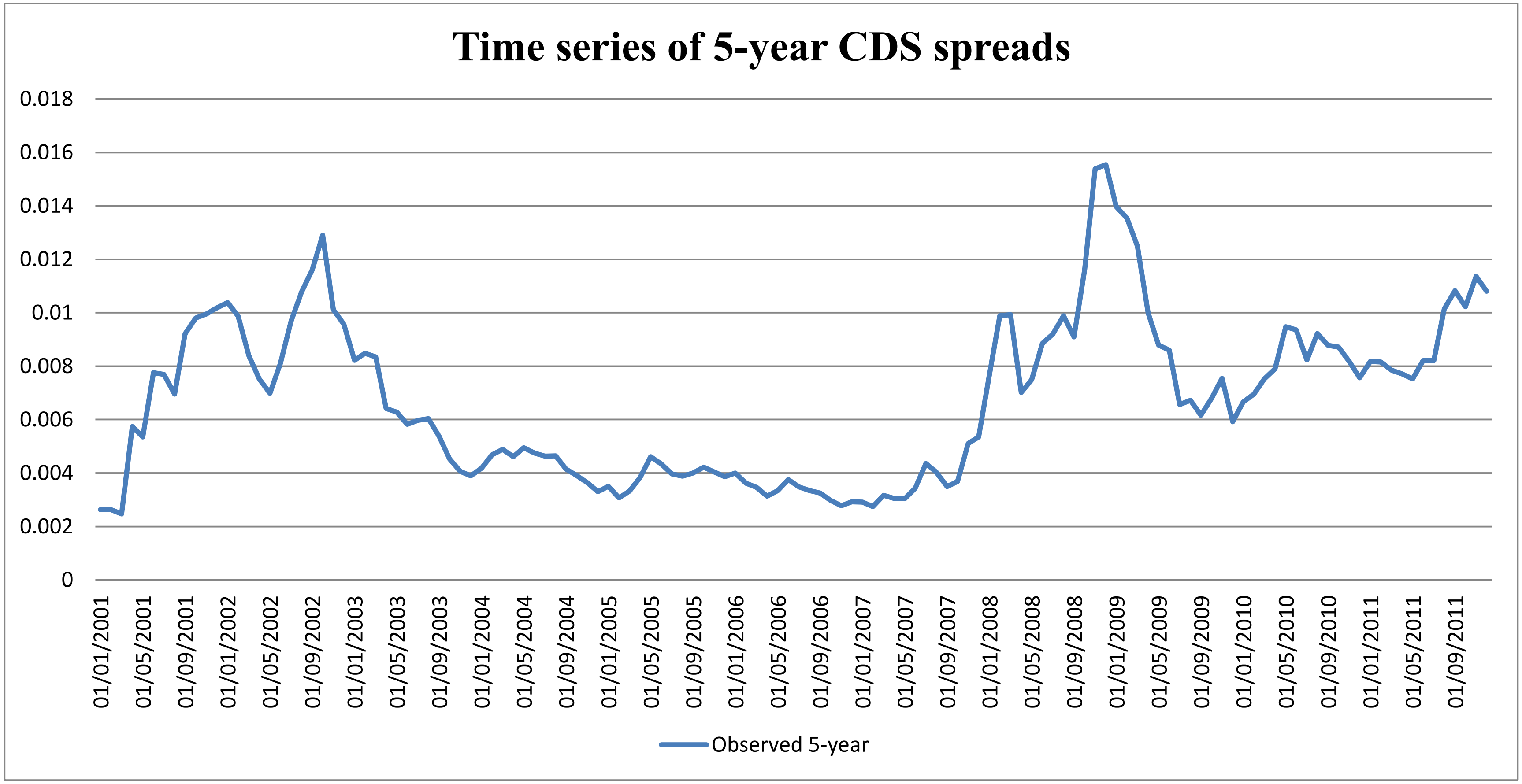

Figure 2: Time series of model calculated 5-year CDS spreads under 7-moments setting.

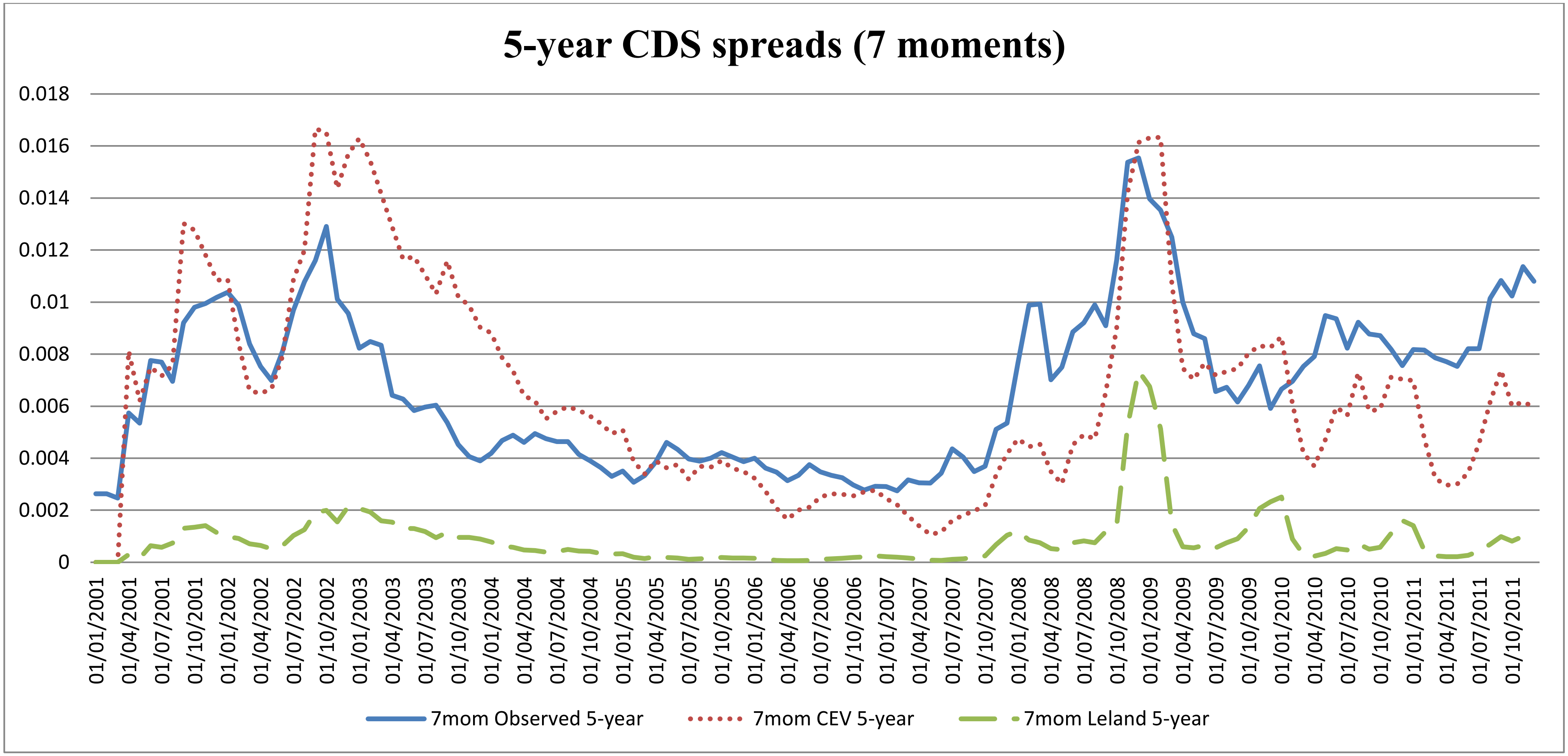

Figure 3: Time series of equity volatility under 7-moments setting.

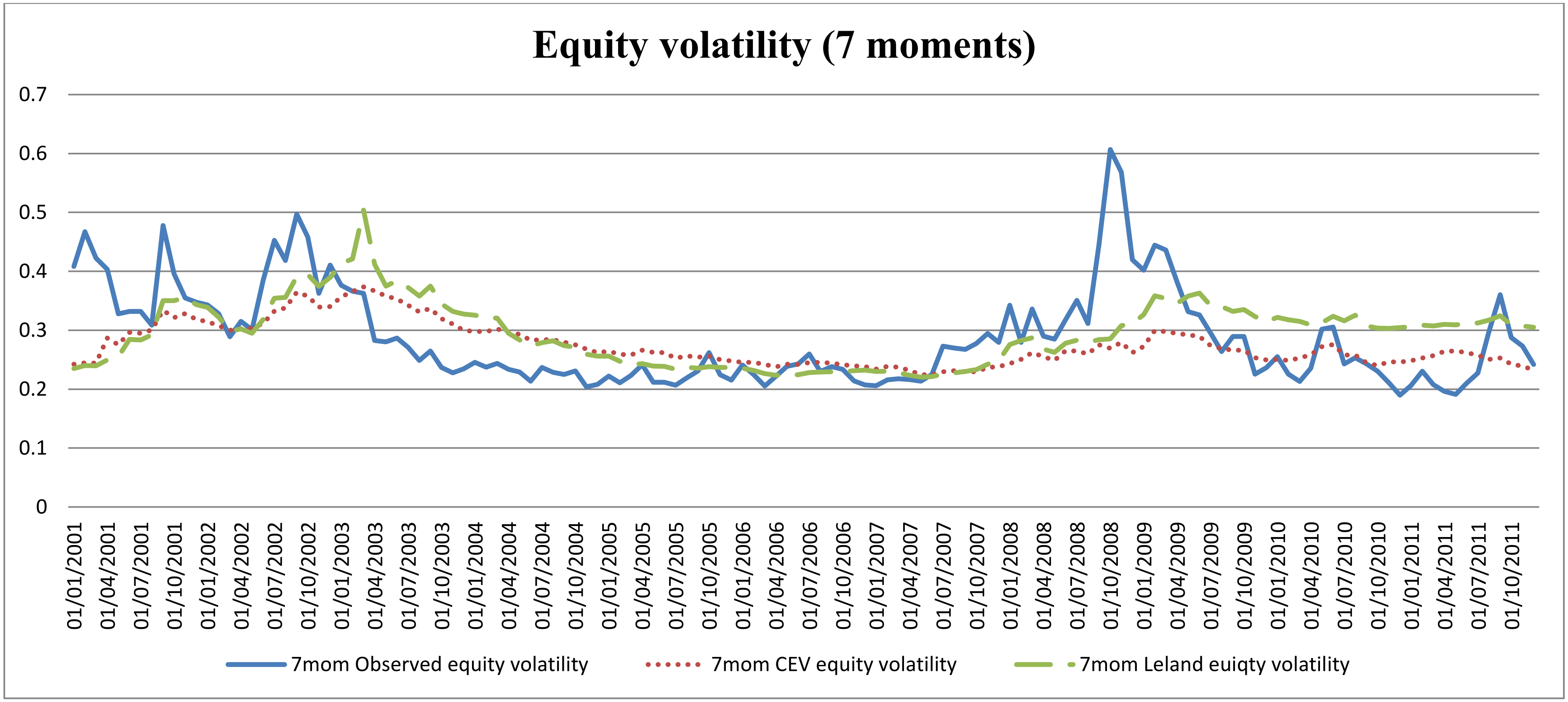

Figure 4: Time series of model calculated 5-year CDS spreads under 9-moments setting.

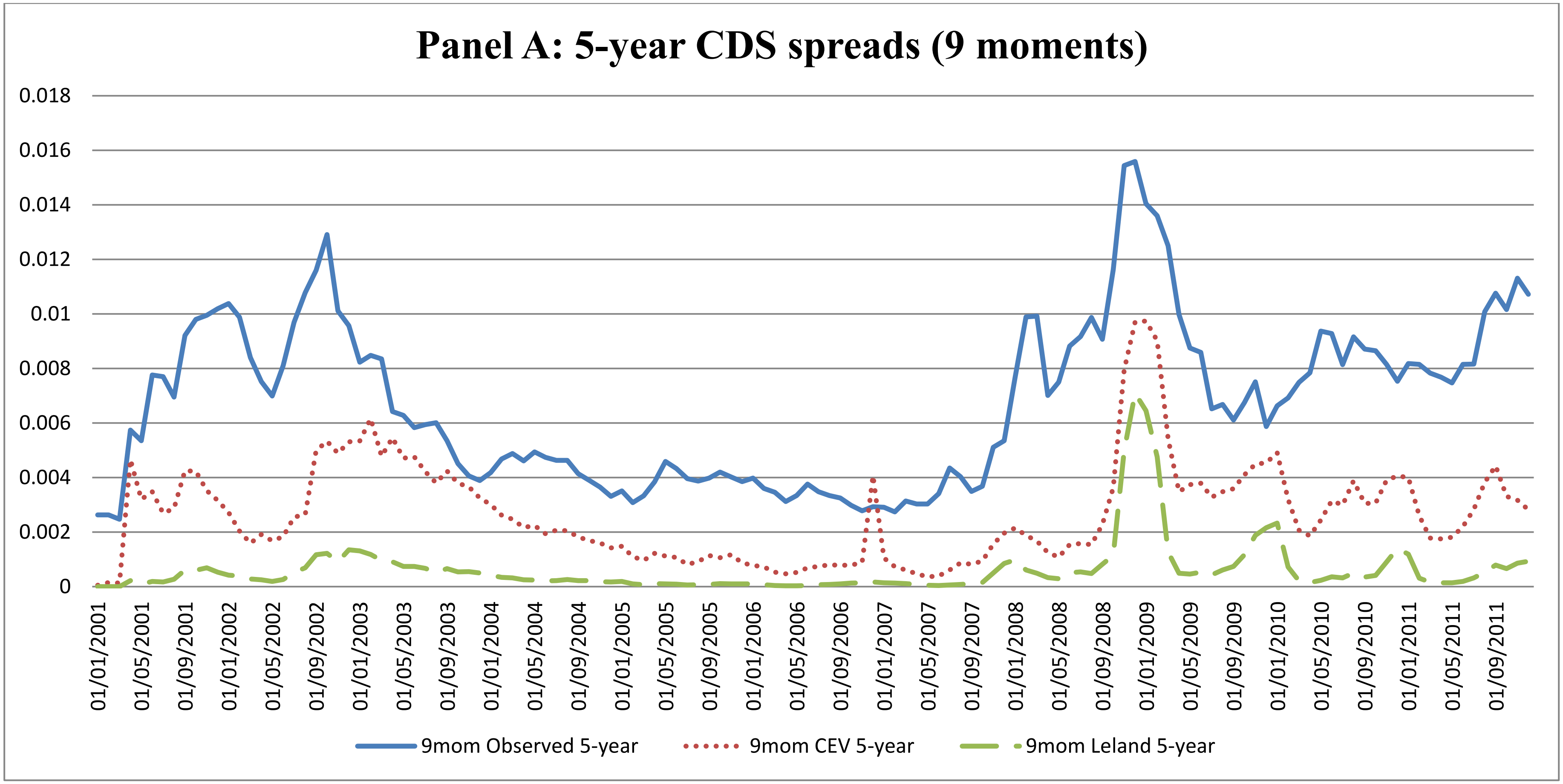

Figure 5: Time series of equity volatility under 9-moments setting.

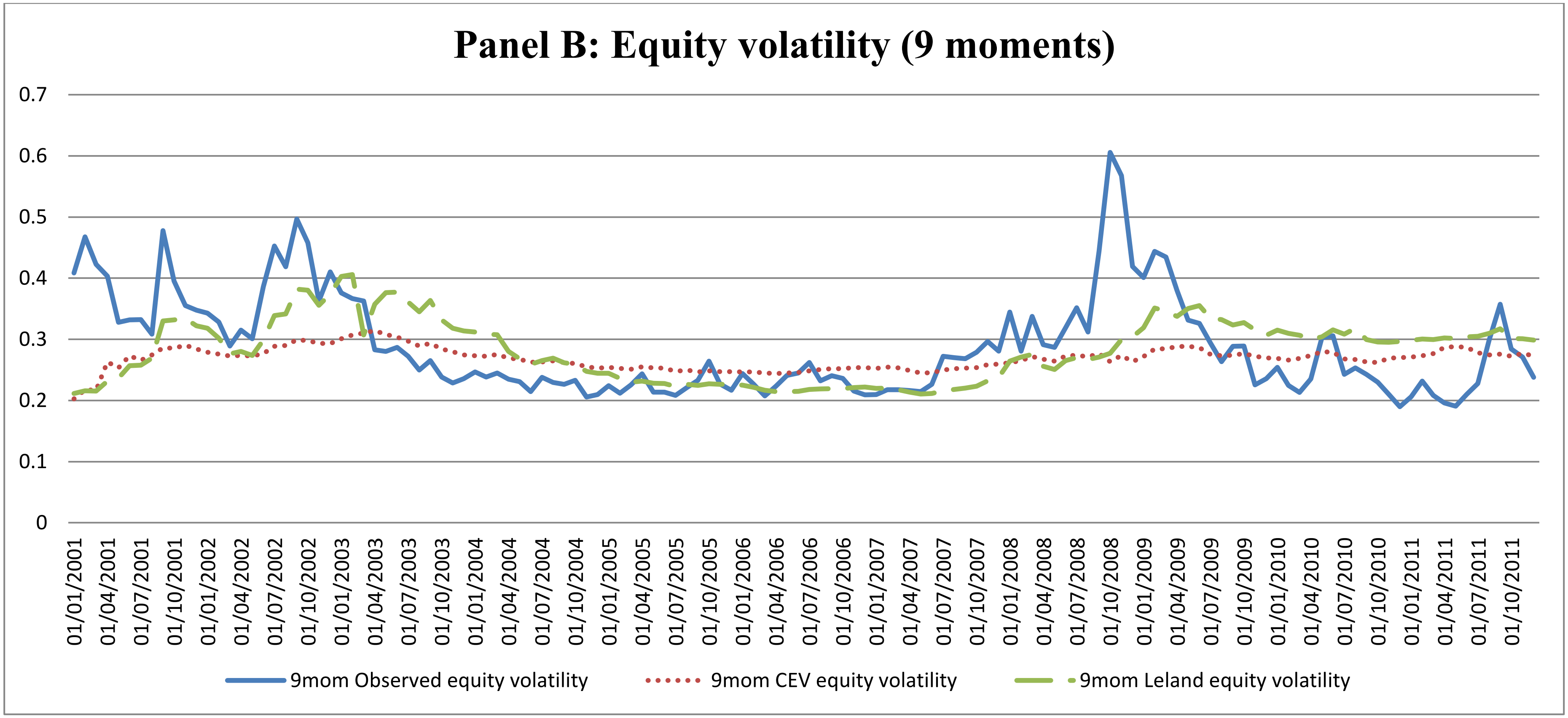

Figure 6: Time series of all scenarios together.